ACTAAP

Arkansas Comprehensive Testing, Assessment, and Accountability Program

Teacher Handbook

Geometry
End-of-Course Examinations

2010-2011 Administrations

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Arkansas Department of Education

Table of Contents

PAGE

Introduction **1**

Scoring Student Responses to Open-Response Items **2**
Reader Training 2
Scoring Procedures 2

Item A—2011 Geometry **4**
Solution and Scoring 6

Item A Sample Responses and Annotations—2011 Geometry **8**
Score Point: 4 8
Score Point: 3 9
Score Point: 2 10
Score Point: 1 11
Score Point: 0 12

Item B—2011 Geometry **13**
Solution and Scoring 14

Item B Sample Responses and Annotations—2011 Geometry **17**
Score Point: 4 17
Score Point: 3 18
Score Point: 2 19
Score Point: 1 20
Score Point: 0 21

Item C—2011 Geometry **22**
Solution and Scoring 24

Item C Sample Responses and Annotations—2011 Geometry **27**
Score Point: 4 27
Score Point: 3 28
Score Point: 2 29
Score Point: 1 30
Score Point: 0 31

Item D—2011 Geometry **32**
Solution and Scoring 33

Item D Sample Responses and Annotations—2011 Geometry **36**
Score Point: 4 36
Score Point: 3 37
Score Point: 2 38
Score Point: 1 39
Score Point: 0 40

Table of Contents

Item E—2011 Geometry **41**
Solution and Scoring 43

Item E Sample Responses and Annotations—2011 Geometry **46**
Score Point: 4 46
Score Point: 3 47
Score Point: 2 48
Score Point: 1 49
Score Point: 0 50

Introduction

The **Arkansas Comprehensive Testing, Assessment, and Accountability Program (ACTAAP)** includes Mid-Year and Spring Geometry End-of-Course Examinations for students completing Geometry or the equivalent for high school graduation credit. The examinations consist of multiple-choice and open-response questions that directly assess student knowledge. The *Arkansas Geometry Mathematics Curriculum Framework* is the basis for the development of the Geometry End-of-Course Examinations.

In January or April 2011, all students who had completed or were completing the required course work for Geometry for high school graduation credit participated in the Mid-Year or Spring Geometry End-of-Course Examination. Results of the Geometry End-of-Course Examinations will be provided to all students, schools, and districts to be used as the basis for instructional change.

This handbook provides information regarding the scoring of student responses to the Geometry open-response items. It describes the scoring procedures and the scoring criteria (rubrics) used to assess student responses. Copies of actual student responses are provided, along with scores given to those responses, to illustrate how the scoring criteria were applied to Geometry open-response items.

Additional information about the Geometry End-of-Course Examination is available through the Arkansas Department of Education. Questions can be addressed to the Assessment Office at 501-682-4558.

Scoring Student Responses to Open-Response Items

The multiple-choice and open-response test items for the Geometry End-of-Course Examination are developed with the assistance and approval of the Geometry Content Advisory Committee. This committee is composed of active Arkansas educators with expertise in Mathematics education. The Geometry Content Advisory Committee develops and reviews multiple-choice and open-response items to ensure that they reflect the *Arkansas Geometry Mathematics Curriculum Framework*.

While multiple-choice items are scored by machine to determine if the student chose the correct answer from four options, open-response items must be scored by trained "readers" using a pre-established set of scoring criteria.

Reader Training

Readers are trained to score only one content area. Qualified readers for the Arkansas scoring will be those with a four-year college degree in English, language arts, education, mathematics, science, or related fields.

Before readers are allowed to begin assigning scores to any student responses, they go through intensive training. The first step in that training is for the readers to read the Geometry open-response items as they appear in the test booklet and to respond—just as the student test takers are required to do. This step gives the readers some insight into how the students might have responded. The next step is the readers' introduction to the scoring rubric. All of the specific requirements of the rubric are explained by the Scoring Director who has been specifically trained to lead the scoring group. Then responses (anchor papers) that illustrate the score points of the rubric are presented to the readers and discussed. The goal of this discussion is for the readers to understand why a particular response (or type of response) receives a particular score. After discussion of the rubric and anchor papers, readers practice scoring sets of responses that have been pre-scored and selected for use as training papers. Detailed discussion of the responses and the scores they receive follows.

After three or four of these practice sets, readers are given "qualifying rounds." These are additional sets of pre-scored papers, and, in order to qualify, each reader must score in exact agreement on at least 80% of the responses and have no more than 5% non-adjacent agreement on the responses. Readers who do not score within the required rate of agreement are not allowed to score the Geometry End-of-Course Examination responses.

Once scoring of the actual student responses begins, readers are monitored constantly throughout the project to ensure that they are scoring according to the criteria. Daily and cumulative statistics are posted and analyzed, and Scoring Directors or Team Leaders reread selected responses scored by the readers. These procedures promote reliable and consistent scoring. Any reader who does not maintain an acceptable level of agreement is dismissed from the project.

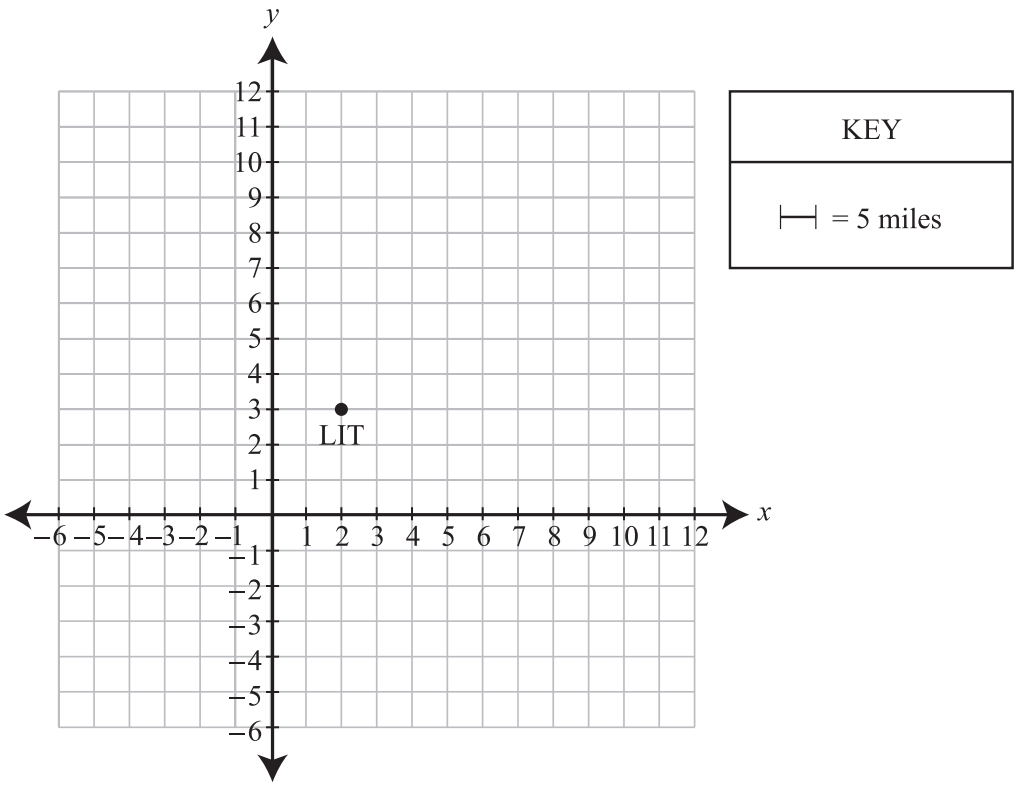
Scoring Procedures

All student responses to the Geometry End-of-Course Examination open-response test items are scored independently by two readers. Those two scores are compared, and responses that receive scores that are non-adjacent (a "1" and a "3," for example) are scored a third time by a Team Leader or the Scoring Director for resolution.

On the following pages, open-response items are presented as they appeared in the *2011 Mid-Year* and *Spring Geometry End-of-Course Examinations*. The specific scoring rubric for each item and annotated responses for each score point of the rubric follows. The goal is for classroom teachers and their students to understand how responses are scored. It is hoped that this understanding will help students see what kind of performance is expected of them on the Geometry End-of-Course Examination.

GEOMETRY RESPONSES

A. An air traffic control system at Little Rock National (LIT) airport, located at (2, 3) on the grid below, uses a radar system that sends out signals to determine the locations of airplanes. This system can detect planes within a circular region having a radius of 35 miles from LIT. Each grid unit represents 5 miles.



An airplane is heading directly toward LIT from the location represented by coordinates (−4, 7) on the grid.

1. Can the plane be detected by the radar? Support your answer with mathematical evidence.

2. The air traffic controller instructs the pilot to begin circling the airport halfway between the airport and her current location. What will be the coordinates of the plane's location when the pilot begins to circle the airport? Show your work or explain your answer.

BE SURE TO LABEL YOUR RESPONSES 1 AND 2.

Item A Scoring Rubric—2011 Geometry

Score	Description
4	The student earns 4 points. The response contains no incorrect work. Parentheses are included if coordinates are listed as an ordered pair in Part 2.
3	The student earns 3 points.
2	The student earns 2 points.
1	The student earns 1 point or some minimal understanding is shown. Ex: Each coordinate is incorrect due to a calculation or copy error in Part 2 The correct procedure is shown for both.
0	The student earns 0 points. No understanding is shown.
B	Blank—No Response. A score of "B" will be reported as "NA." (No attempt to answer the item. Score of "0" assigned for the item.)

SOLUTION AND SCORING

Part	Points
1	**2 points possible** 2 points: Correct answer of "No" (or equivalent) Correct procedures are shown and/or explained Give credit for the following or equivalent**:** *Note: Distance may be rounded to 10ths place or smaller at any level.* • Uses Distance Formula: Ex: "No, the plane cannot be detected. The plane's distance from the airport is: $\sqrt{(2-(-4))^2+(3-(7))^2} = \sqrt{36+16} = \sqrt{52} \approx 7.2$ units 7.2 units is 36 miles away, so the plane is too far." Ex: "No because $\sqrt{20^2+30^2} = 36.05$." • Uses Pythagorean Theorem: Ex: $4^2+6^2=c^2$ $52=c^2$ $x=7.21$ "7.21 ≥ 7, so plane can't be detected." *(35 miles = 7 units not required)* **OR** 1 point: Give credit for the following: *Note: Labels of "miles" and/or "units" may be incorrect or missing.* • Answer of "No" is incorrect or missing Work is correct and complete or • Work contains a calculation, copy, counting or rounding error *Note: Writing down given coordinate(s) incorrectly will be considered a copy error.* Correct procedures are shown and/or explained Correct answer based on error is given or • Correct answer of "No" or equivalent Work is incomplete, but some correct procedure is shown Ex: "No because it's about 7.21 units away which is too far." *(work to find 7.21 is missing)*

Part	Points
2	**2 points possible** 2 points: Correct coordinates: (–1, 5) (Parentheses are required for a "4") or $x = -1$ and $y = 5$ (or correct coordinates based on a copy error from Part 1) Correct procedure shown and/or explained Give credit for the following or equivalent: • Uses Midpoint Formula: Ex: $\left(\frac{2+(-4)}{2}, \frac{3+7}{2}\right) = (-1, 5)$ • Uses Slope and horizontal and vertical distances: Ex: (–1, 5) "The slope is $-\frac{2}{3}$ so if I go up 2 and left 3 from LIT, I will reach the midpoint. To get from the midpoint to the plane, I go up 2 and left 3." **OR** 1 point: Give credit for the following: • Correct coordinates Work is incomplete or missing or • Coordinate(s) incorrect due to a calculation, counting or visible copy error(s) Correct procedure is shown and/or explained

SCORE POINT: 4

Part 1		**Points**
Correct answer with correct procedure:	"The plane can't be detected" Uses Distance Formula: $\sqrt{(-4-2)^2+(7-3)^2}\ \ldots = 7.21$ $(7.21)(5) = 36.05$	**2**

Part 2		
Correct coordinates with correct procedure:	(–1, 5) Uses Midpoint Formula: $\left(\frac{-4+2}{2}\right), \left(\frac{7+3}{2}\right)$	**2**
	Total Points	**4**

1) The plane can't be detected because it is 1+ mile too far to be detected by the radar.

$d = \sqrt{(-4-2)^2 + (7-3)^2}$

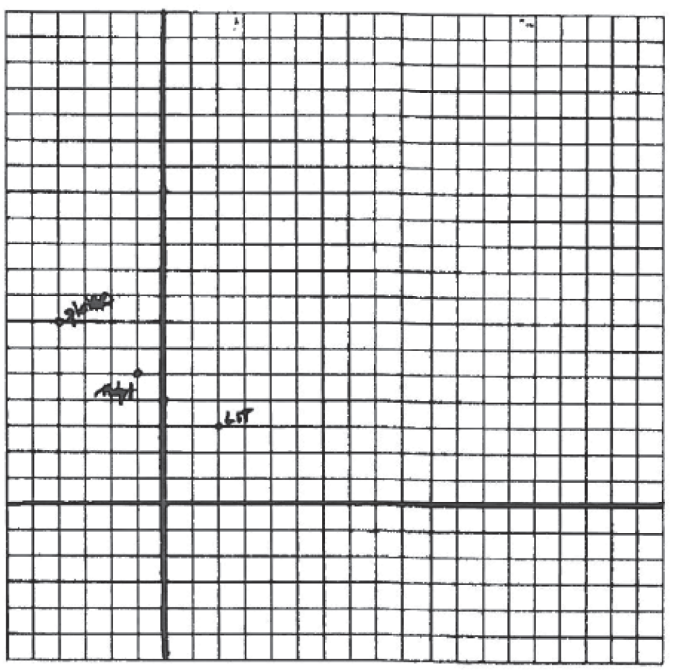
$d = \sqrt{36 + 16}$

$d = 7.21$ units

real $d = (7.21)(5)$

real $d = 36.05$ mi

2) The pilot's location when ~~she~~ begins to circle the airport is (-1, 5)



$mdpt = \left(\frac{x_2 + x_1}{2}\right), \left(\frac{y_2 + y_1}{2}\right)$

$mdpt = \left(\frac{-4+2}{2}\right), \left(\frac{7+3}{2}\right)$

$mdpt = (-1, 5)$

SCORE POINT: 3

Part 1		**Points**
Correct answer based on calculation error with correct procedure:	"Yes it can" Uses Distance Formula: $\sqrt{(-4-2)^2+(7-3)^2} \ldots = \sqrt{(-2)^2\ (calc.er)+(4^2)}$ $(4.47)(5) = 22.4$	1

Part 2		
Correct coordinates with correct procedure:	(–1, 5) Uses Midpoint Formula: $\left(\frac{-4+2}{2}\right), \left(\frac{7+3}{2}\right)$ etc.	2
	Total Points	**3**

① $d = \sqrt{(x_2-x_1)^2+(y_2-y_1)^2}$

$d = \sqrt{(-4-2)^2+(7-3)^2}$

$= \sqrt{(-2)^2+(4)^2}$

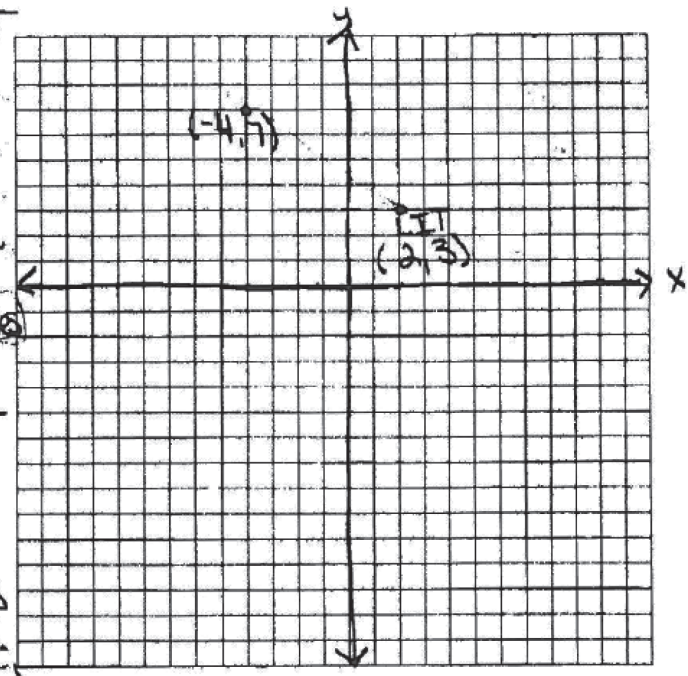
$= \sqrt{4+16}$

$= \sqrt{20}$

$= 4.47$

$4.47 \cdot 5 = 22.4$ miles

Yes it can because it can detect up to 35 miles away and this plane in only 22.4 miles away from LII.



② $m = \left(\frac{x_2+x_1}{2}, \frac{y_2+y_1}{2}\right)$

$m = \left(\frac{-4+2}{2}, \frac{7+3}{2}\right)$

$= \left(\frac{-2}{2}, \frac{10}{2}\right)$

(-1,5)

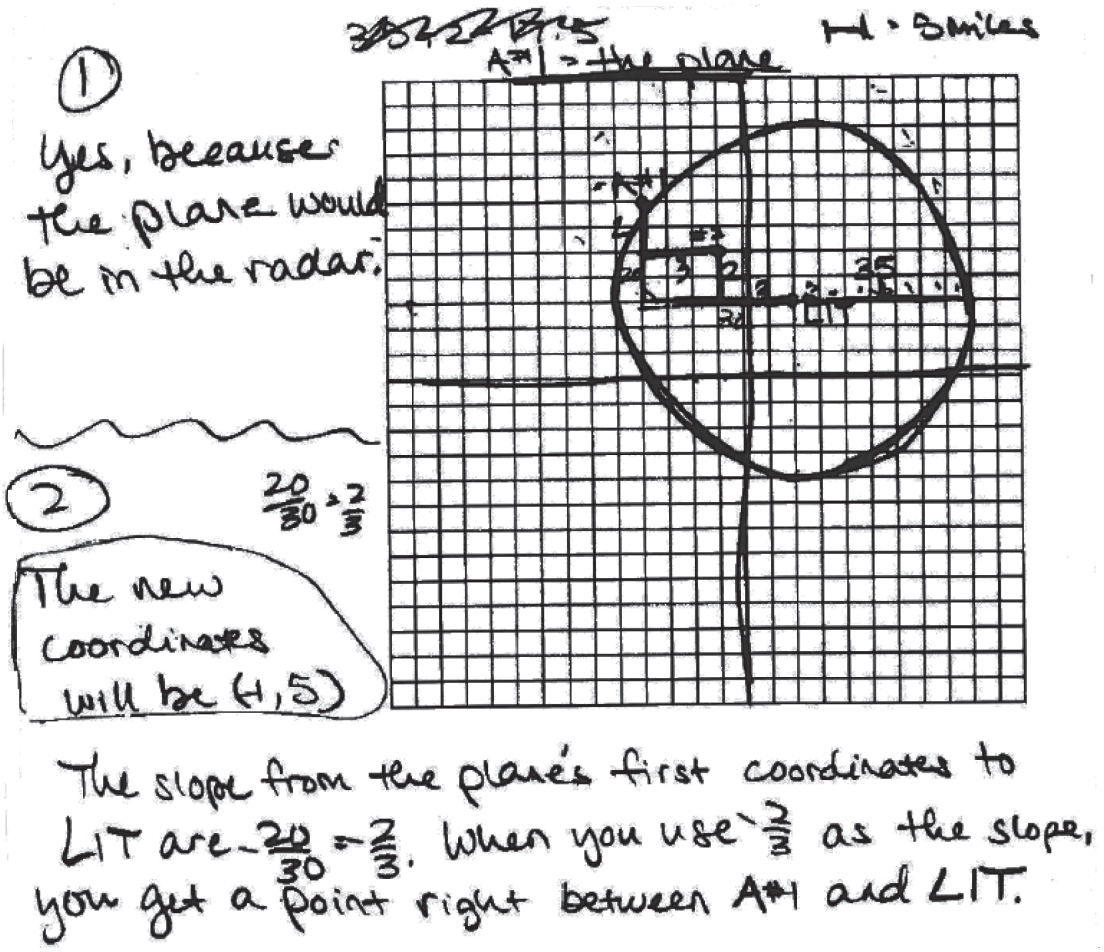
She will be at (-1,5) when she begins to circle the airport.

SCORE POINT: 2

Part 1		**Points**
Incorrect answer with missing procedure:	"Yes,"	–

Part 2

Correct coordinates with correct procedure:	(–1, 5) Uses Slope and Horizontal and Vertical distances: Slope is $-\frac{2}{3}$ Shows "down 2 over 3, down 2 over 3" on diagram	**2**
	Total Points	**2**



SCORE POINT: 1

Part 1		**Points**
Incorrect answer with incorrect procedure:	"Yes" Attempts to write the equation of the circle	–

Part 2

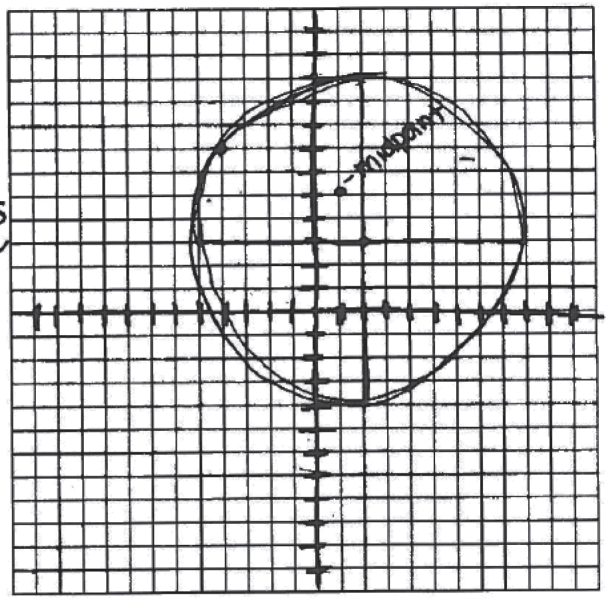
1 Correct coordinate with correct procedure:	(1, 5) (x-coordinate incorrect due to calc. error) Uses Midpoint Formula: $\frac{(-4+2)}{2}$ $\frac{(7+3)}{2}$	**1**
	Total Points	**1**

1) Yes the plane will be detected because the point lies within the region of the circle $(x-2)^2(y-3)^2=\sqrt{35}$

2) $\frac{(x_2+x_1)}{2}\frac{(y_2+y_1)}{2}$

$\frac{(-4+2)}{2}$ $\frac{(7+3)}{2}$

$(1,5)$



When the pilot begins to circle she will be on the point (1,5) because (1,5) is the midpoint (the middle) of the other points (-4,7) & (2,3). The midpoint formula is $\frac{(x_2+x_1)}{2}$ $\frac{(y_2+y_1)}{2}$

$35 \div 5 = 7$

$r = \boxed{7 \text{ units}}$

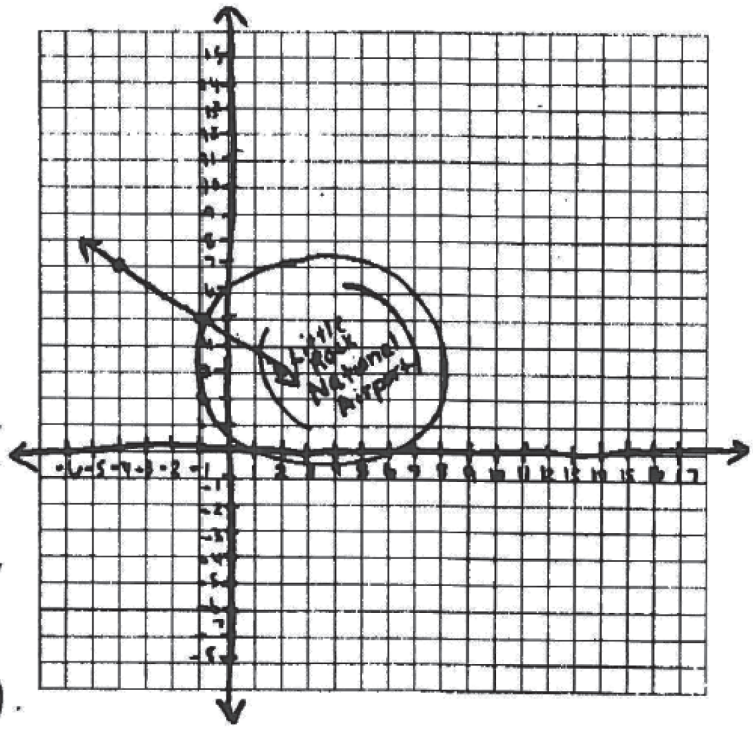
SCORE POINT: 0

Part 1		**Points**
Incorrect answer with missing procedure:	"The plane can be detected"	–

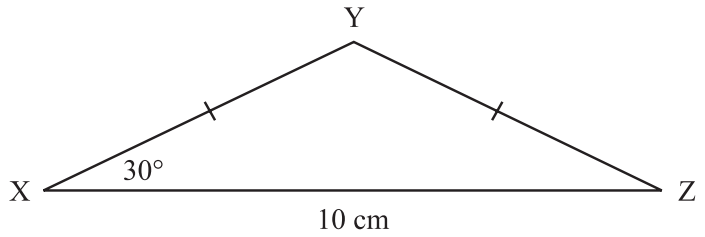
Part 2		
Incorrect coordinates with missing procedure:	(5, –1)	–
	Total Points	**0**

1) The plane can be detected by the radar because it is within 30 miles of the Little Rock National Airport.

2) When the pilot begins to circle the airport, her current location will be at (5,-1).



B. ΔXYZ is shown below.



1. Determine $m\angle Y$. Show your work or explain how you found your answer.

2. An altitude is drawn from point Y to $\overline{XZ}$, and the point where the altitude intersects $\overline{XZ}$ is labeled W. What is YW? Give your answer in simplest radical form or round your answer to the nearest hundredth. Show your work or explain how you found your answer.

3. Determine the area of ΔXYZ. Give your answer in simplest radical form or round your answer to the nearest hundredth. Show your work or explain how you found your answer.

BE SURE TO LABEL YOUR RESPONSES 1, 2, AND 3.

Item B Scoring Rubric—2011 Geometry

Score	Description
4	The student earns 4 points. The response contains no incorrect work. The response contains: Correct label of "degrees" or "°" in Part 1 Correct label of "cm" in Part 2 Correct label of "sq. cm" in Part 3
3	The student earns **3–3½** points.
2	The student earns **2–2½** points.
1	The student earns **½–1½** points or some minimal understanding is shown.
0	The student earns **0** points. No understanding is shown.
B	Blank—No Response. A score of "B" will be reported as "NA." (No attempt to answer the item. Score of "0" assigned for the item.)

SOLUTION AND SCORING

Part	Points
1	**1 point possible** 1 point: Correct measure: 120 (degrees) Correct procedure is shown and/or explained Give credit for the following or equivalent: Ex: Since XY = YZ, m∠Z = m∠X = 30°. so m∠Y = 180 – 30 – 30 = 120 Ex: 180 – (2*x*30) = 120 Ex: The triangle is isosceles, so if the altitude is drawn from Y to $\overline{XZ}$ it will be perpendicular and form 90° angles. Therefore, each will be a 30 – 60 – 90 triangle, and m$\angle Y = 2(60) = 120$ degrees. **OR** ½ point Give credit for the following or equivalent: • Correct answer Procedure is incomplete or missing or • Answer is incorrect due to calculation or copy error Correct procedure is shown and/or explained
2	**2 points possible** 2 points: Correct length: $\frac{5\sqrt{3}}{3}$ or 2.89 (cm) Correct procedure is shown and/or explained Give credit for the following or equivalent: • Uses properties of Special Right Triangles: Ex: ΔYWZ is a 30-60-90 triangle $5 = a\sqrt{3}$ $a = \frac{5}{\sqrt{3}} = \frac{5\sqrt{3}}{3}$ Ex: $a\sqrt{3} = 5$ $a = \frac{5}{1.732} = 2.89$ Ex: $\frac{1}{x} = \frac{\sqrt{3}}{5}$ $x\sqrt{3} = 5$ $x = 2.89$

Part	Points
2	**2 points possible (continued)** • Uses Trig. Functions: Ex: $\tan 30° = \frac{YW}{5}$ YW = 5 tan 30° = 2.886 = 2.89 Ex: $\tan 30° = \frac{x}{5}$ $x = \frac{5\sqrt{3}}{3}$ • Uses Law of Sines: Ex: $\frac{\sin 30°}{y} = \frac{\sin 60°}{5}$ etc. **OR** 1½ points: Answer is correctly rounded or truncated to a place other than the 100ths place or is not left in simplest radical form Correct procedure is shown and/or explained **OR** 1 point: • Correct answer of $\frac{5\sqrt{3}}{3}$ or 2.89 Procedure is incomplete or missing or contains incorrect trig. notation or • Answer is incorrect due to calculation, copy or rounding error Correct procedure is shown and/or explained **OR** ½ point: Answer is rounded, truncated etc. Some support is given

Part	Points
3	**1 point possible** 1 point Correct answer (based upon input): 14.43 or 14.45 (sq. cm) (or 14.46 with correct procedure of finding area of 2 triangles) (or correct answer rounded to 100ths place based on an incorrect answer in Pt. 2) Correct procedure is shown and/or explained Give credit for the following or equivalent: Ex: $A = \frac{10(2.89)}{2} = 14.45$ Ex: $A = \frac{1}{2}(10)\left(\frac{5\sqrt{3}}{3}\right) = 14.43 \quad \left(or\ \frac{25\sqrt{3}}{3}\right)$ **OR** ½ point: Give credit for the following or equivalent: • Correct answer Procedure is incomplete or missing or • Answer is incorrect due to calculation, copy or rounding error and/or answer is not rounded to the nearest 100ths place Correct procedure is shown and/or explained

SCORE POINT: 4

Part 1

		Points
Correct measure with correct procedure:	120 (degrees) 30 + 30 = 60 180 – 60 = 120	1

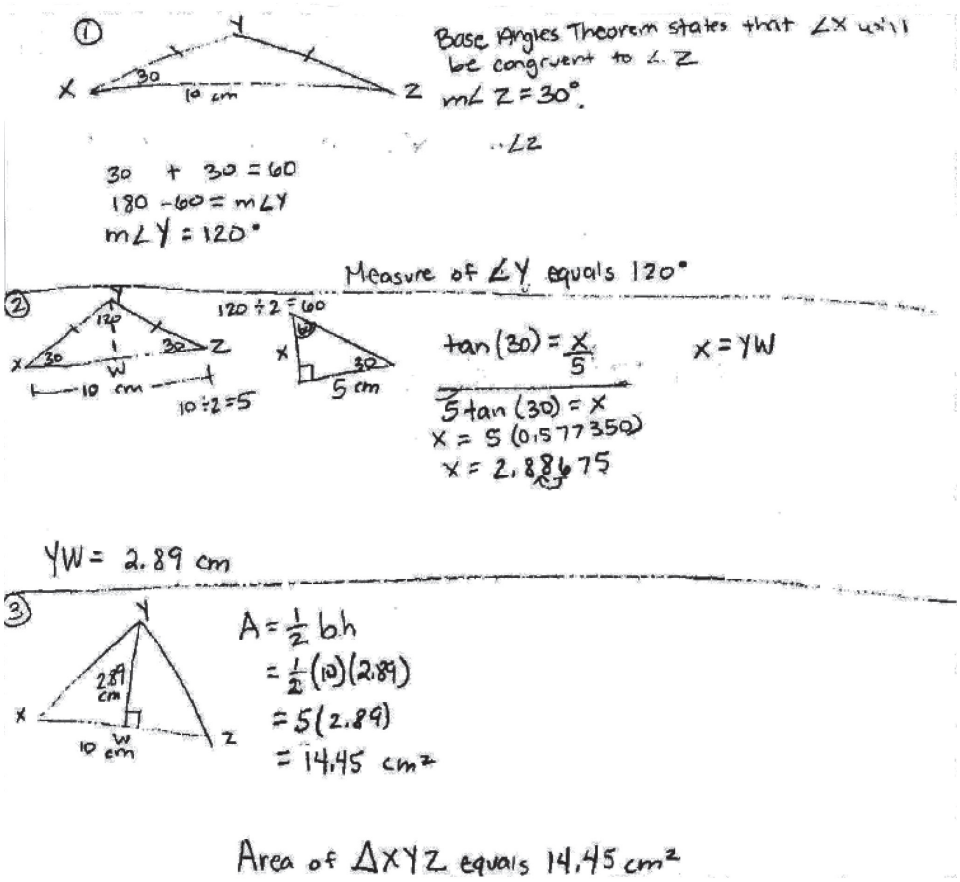
Part 2

Correct length with correct procedure:	2.89 (rounded to nearest 100th) (cm) Sets up equation using trig. functions and solves: $\tan(30) = \frac{x}{5}$ etc.	2

Part 3

Correct area with correct procedure:	14.45 (rounded to nearest 100th) (sq. cm) Uses formula to find the area of the triangle	1
	Total Points	**4**

Correct labels are included in Parts 1, 2, and 3.



SCORE POINT: 3

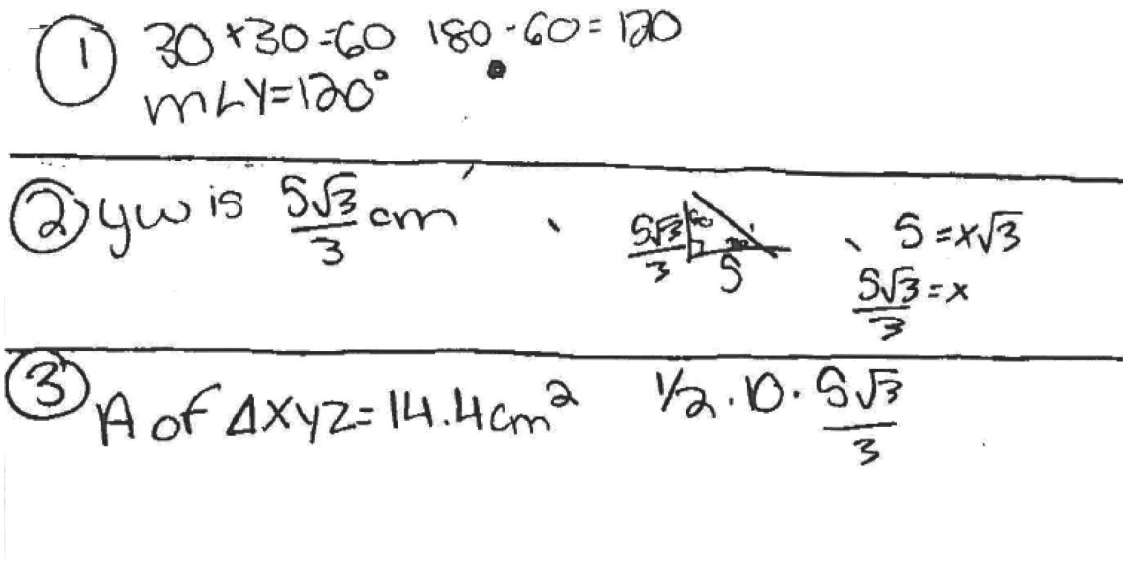
Part 1		**Points**
Correct measure with correct procedure:	120 (degrees) 30 + 30 = 60 180 – 60 = 120	**1**

Part 2

Correct length with correct procedure:	$\frac{5\sqrt{3}}{3}$ (simplified radical form) (cm) Uses properties of special right triangles	**2**

Part 3

Incorrect area with correct procedure:	14.4 (not rounded to nearest 100th) (sq. cm) Uses formula to find the area of the triangle	½
	Total Points	**3½**

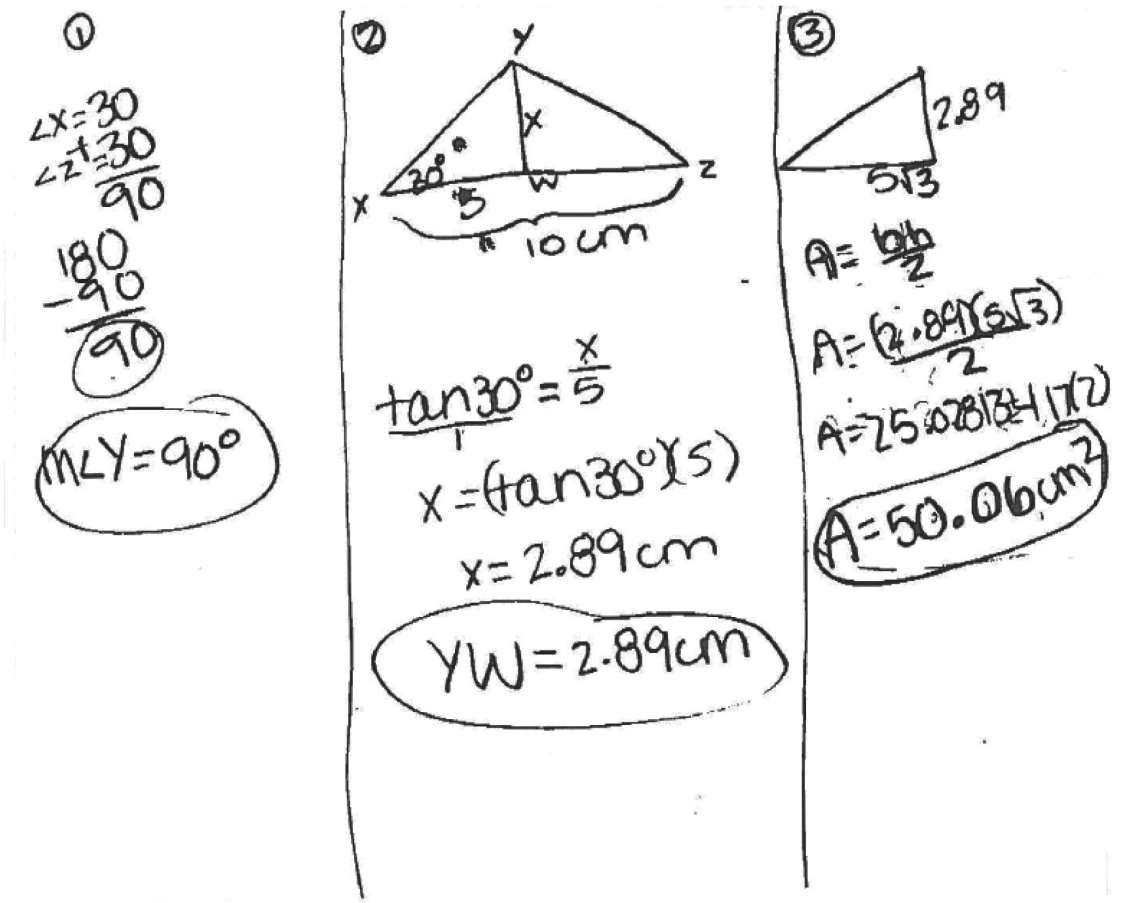


SCORE POINT: 2

Part 1		**Points**
Incorrect measure with correct procedure:	90 (due to calculation error) (degrees) 30 + 30 = 90 (calc. er.) 180 – 90 = 90	½

Part 2		
Correct length with correct procedure:	2.89 (rounded to nearest 100th) Sets up equation using trig. functions and solves	2

Part 3		
Incorrect area with incorrect procedure:	50.06 Uses $5\sqrt{3}$ for base of triangle	–
	Total Points	**2½**



SCORE POINT: 1

Part 1		**Points**
Incorrect measure with incorrect procedure:	60 (degrees) 120 – 60 = 60	–

Part 2		
Incorrect length with incorrect procedure:	15.37 Uses "10" for adjacent length	–

Part 3		
Correct area based on Part 2 with correct procedure:	76.85 (rounded to nearest 100th) Uses formula to find the area of the triangle	**1**
	Total Points	**1**

1) $\angle X = 30^\circ$ $\quad$ $120^\circ - 60^\circ = 60$

$\angle X = \angle Z$ $\quad$ $\angle Y = 60^\circ$

$\angle Z = 30^\circ$

2) $\tan^{-1}(30) = \frac{x}{10}$ $\quad$ $\overline{YW} = 15.37\text{cm}$

$x = \tan^{-1}(30) \cdot 10$

$x = 15.37$

3) $\frac{B \cdot h}{2}$ $\quad$ $\frac{10\text{cm} \cdot 15.37\text{cm}}{2}$

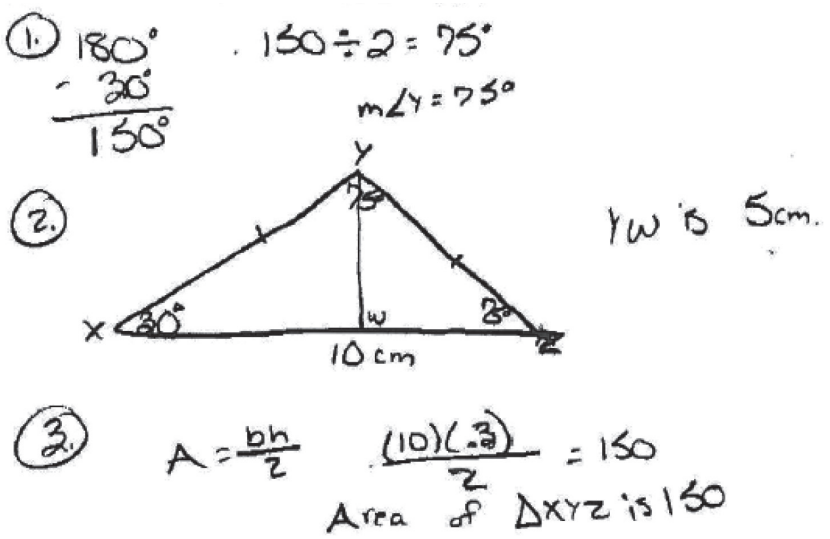
$\frac{153.7}{2} = 76.85\text{cm}^2$ is the area of $\triangle XYZ$

SCORE POINT: 0

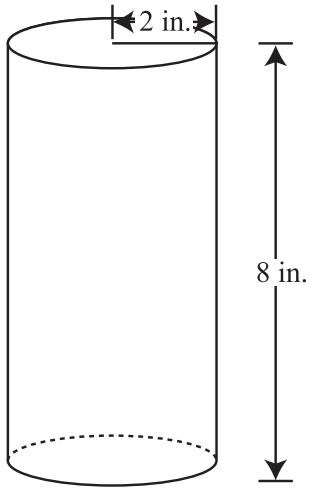
Part 1		**Points**
Incorrect measure with incorrect procedure:	75 180 – 30 = 150 etc.	–

Part 2		
Incorrect length with missing procedure:	5	–

Part 3		
Incorrect area based on Part 2 with incorrect procedure:	150 Uses formula for the area of a triangle, with calculation error, but doesn't use *h* from Part 2	–
	Total Points	**0**



C. Double Gold Food Company currently packages its oatmeal in cardboard cylinders with the dimensions shown below.



Double Gold Food plans to increase the volume of the containers 25%.

1. If Double Gold Food decides to change the height of the current container but keep the radius the same, what will be the height of the new container? Show your work or explain how you found your answer.

2. If Double Gold Food decides to change the radius of the current container but keep the height the same, what will be the radius of the new container? Round your answer to the nearest hundredth of an inch. Show your work or explain how you found your answer.

3. The amount of materials needed to create the container depends on its surface area. Determine which option, increasing the height only or increasing the radius only, results in a cylindrical container with the **least** surface area? Show your work or explain how you found your answer.

BE SURE TO LABEL YOUR RESPONSES 1, 2, AND 3.

Item C Scoring Rubric—2011 Geometry

Score	Description
4	The student earns 5 points. The response contains no incorrect work. The Label of "inches" is used in Part 1.
3	The student earns **3½–4½** points.
2	The student earns **2–3** points.
1	The student earns **½–1½** points or some minimal understanding is shown.
0	The student earns **0** points. No understanding is shown.
B	Blank—No Response. A score of "B" will be reported as "NA." (No attempt to answer the item. Score of "0" assigned for the item.)

SOLUTION AND SCORING

Note: Do not deduct for internal work that contains early-rounded or truncated values. Many students write these values for the sake of brevity, but use the exact value in the calculator to find their final answer.

Note: Steps may be combined, resulting in incorrect internal work, except at the "4" level.

5 points possible:

Part	Points
1	**1½ points possible** **Note: Work may include values that have been truncated or rounded early (except using Guess & Check), resulting in answers that are not exact.** 1½ points: Correct height: 10 (Accept $9.9 - 9.\overline{9}$) (in) Correct procedure shown and/or explained Give credit for the following or equivalent: • $V = (3.14)(2^2)(8) = 100.48$ $(.25)(100.48) = 25.12$ New $V = 125.6$ $125.6 = (3.14)2^2h \rightarrow 40 = 4h \rightarrow 10 = h$ • V (new container): $(1.25)\pi(2^2)8 = 40\pi = \pi(2^2)h \rightarrow 10 = h$ • V (current container): $V = \pi r^2 h = \pi(2)^2(8) = 32\pi = 100.530...$ V (new container): $1.25 \times 100.530... = 125.663...$ $\pi(2^2)h = 125.663...$ $h = 9.99$ • Guess and Check: (answer and work must be exact) Ex: $V = (1.25)(3.14)(2^2)(8) = 125.6$ $(3.14)(2^2)(10) = 125.6 \rightarrow h = 10$ • $h = 8 + .25(8) = 10$ *(volumes are not required with this method)* **OR** 1 point: • Height is correct Work is incomplete or • Height is incorrect *Work contains a calculation, copy or an obvious rounding error* Correct procedure is shown and/or explained **OR** ½ point: • Height is correct Work is missing

Part	Points
2	**1½ points possible** **Note: Work may include values that have been truncated or rounded early (except using Guess & Check), resulting in answers that are not exact.** 1½ points: Correct radius: 2.24 (in) *(or correct r rounded to the nearest 100th based on V and/or work in Pt. 1)* Correct procedure shown and/or explained Give credit for the following or equivalent: • V (new container): $\pi r^2 8 = 125.663...$ $5 = r^2$ $\sqrt{5} = r = 2.23606... \rightarrow 2.24$ **OR** 1 point: • Radius is correct *(or correct r rounded to the nearest 100th based on V and/or work in Pt. 1)* Work is incomplete or • Radius is incorrect *Work contains a calculation, copy or an obvious rounding error and/or is not rounded to the nearest 100th* Correct procedure is shown and/or explained **OR** ½ point: • Radius is correct *(or correct r rounded to the nearest 100th based on V and/or work in Pt. 1)* Work is missing
3	**2 points possible** **Note: No deduction is made for the rounding or truncating of SA's to any position in Part 3** 2 points: Correct answer: Increasing the radius *(or correct answer based on incorrect answer(s) and Parts 1 and/or 2)* Correct procedure is shown and/or explained Give credit for the following or equivalent: • SA of the cyclinder with the increased height: **(Accept 150-151)** Ex: $SA = 2\pi rh + 2\pi r^2 = 2\pi(2)(10) + 2\pi(2)^2 \approx 150.8$ SA of the cylinder with the increased radius: **(Accept 143-144.13)** Ex: $SA = 2\pi rh + 2\pi r^2 = 112.59 + 31.52 \approx 144.11$ So the surface area would be less if you increase the radius. • $SA = 2\pi r^2 + 2\pi rh = 2\pi r(r + h)$ Increase height: $SA = 2\pi \cdot 2(2 + 10) = 4\pi \cdot 12 = 48\pi$ Increase radius: $SA = 2\pi(2.24)(2.24 + 8) = 45.8752\pi$ Therefore the SA is smaller if you increase *r*.

Part	Points
3	**2 points possible (continued)** **OR** 1½ points: • Both Surface Areas are correctly found with work shown and/or explained Answer is missing **OR** 1 point: • Both Surface Areas are correct with no work shown or explained Answer is correct or • Both Surface Areas are correct with work shown and/or explained Answer is incorrect or • 1 Surface Area is correct 1 Surface Area is incorrect due to a calculation, copy or rounding error Correct procedure is shown and/or explained Answer is correct based on error **OR** ½ point: • Both Lateral Surface Areas are correct (top and bottom are omitted) Work is shown and/or explained Answer is correct Ex: SA = $2\pi rh$ = 2(3.14)(2)(10) = 125.6 SA = $2\pi rh$ = 2(3.14)(2.24)(8) = 112.5376 Increasing the radius results in a smaller SA or • Correct answer based on 2 incorrect total surface areas due to calculation error(s) Correct procedure is shown and/or explained

SCORE POINT: 4

Part 1		**Points**
Correct height correct & complete procedure:	10 (in) Finds V of new cylinder using 3.14: (125.6) Sets up equation and solves for h: $125.6 = \pi(2^2)(h)$ *etc.*	**1½**

Part 2

Correct radius correct & complete procedure:	2.24 *(rounded to nearest 100th)* Sets up equation and solves for r: $125.6 = \pi(r^2)(8)$ *etc.*	**1½**

Part 3

Correct answer with correct & complete procedure:	Increasing the radius SA (h = 10) = 125.6 + 25.12 = 150.72 SA (r = 2.24) = 112.54 + 31.51 = 144.05	**2**
	Total Points	**5**

Label of "inches" is included in Part 1

1.) current $V = \pi 2^2(8)$
$= 100.48 \text{ in.}^3$
increased $V = 100.48 \times 25\%$
$= 25.12$
$= 100.48 + 25.12$
$= 125.6 \text{ in.}^3$
$125.6 = \pi 2^2 h$
$\frac{125.6}{12.56} = \frac{12.56h}{12.56}$
$10 = h$
The height of the new container is 10 inches.

2.) $125.6 = \pi r^2 8$
$\frac{125.6}{25.12} = \frac{25.12r^2}{25.12}$
$\sqrt{5} = \sqrt{r^2}$
$2.24 = r$
The radius of the new container is 2.24 inches.

3.) $SA = 2\pi rh + 2\pi r^2$

increasing height	increasing radius
$SA = 2\pi r(10) + 2\pi r^2$	$SA = 2\pi r(8) + 2\pi 2.24^2$
$= 125.6 + 25.12$	$= 112.54 + 31.51$
$= 150.72 \text{ in}^2$	$= 144.05 \text{ in}^2$

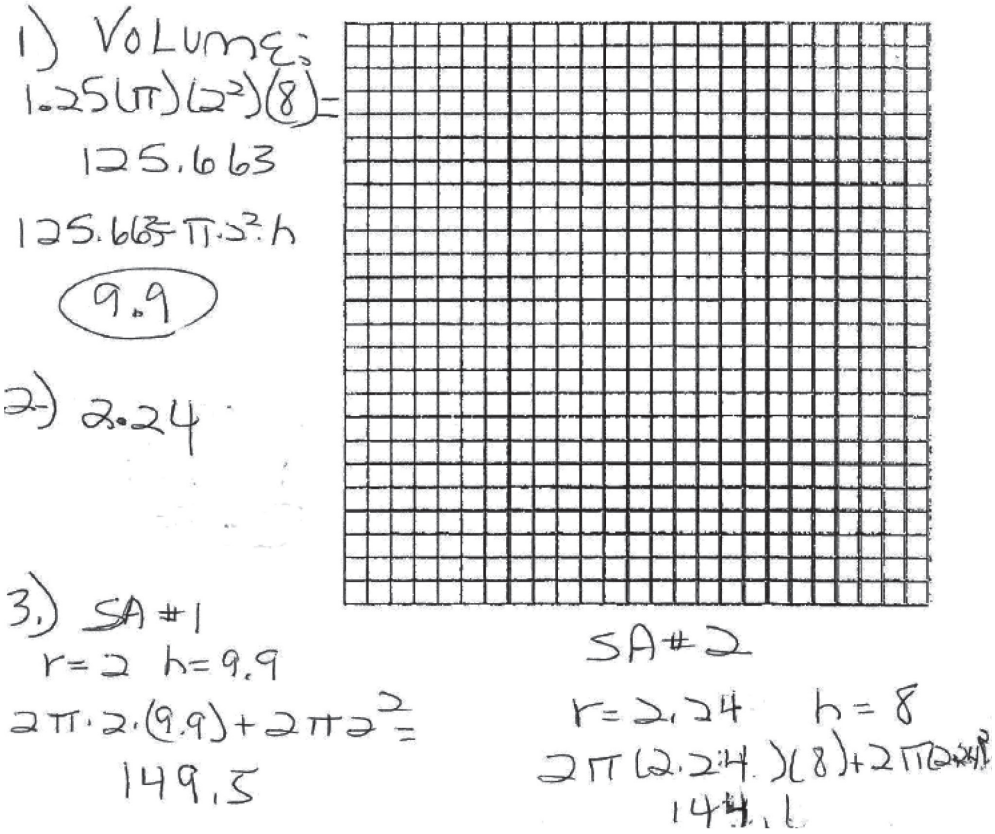
Increasing the radius only results in a container with the least surface area.

SCORE POINT: 3

Part 1		**Points**
Correct height correct & complete procedure:	9.9 Finds V of new cylinder using π key: (125.663) Sets up equation and solves for h: $125.66 = \pi 2^2 h$	**1½**

Part 2		
Correct radius missing procedure:	2.24	**½**

Part 3		
Missing answer with correct & complete procedure based on Parts 1 & 2:	$SA\ (h = 9.9) = 2\pi(2)(9.9) + 2\pi 2^2 = 149.5$ $SA\ (r = 2.24) = 2\pi(2.24)(8) + 2\pi(2.24)^2 = 144.1$	**1½**
	Total Points	**3½**



SCORE POINT: 2

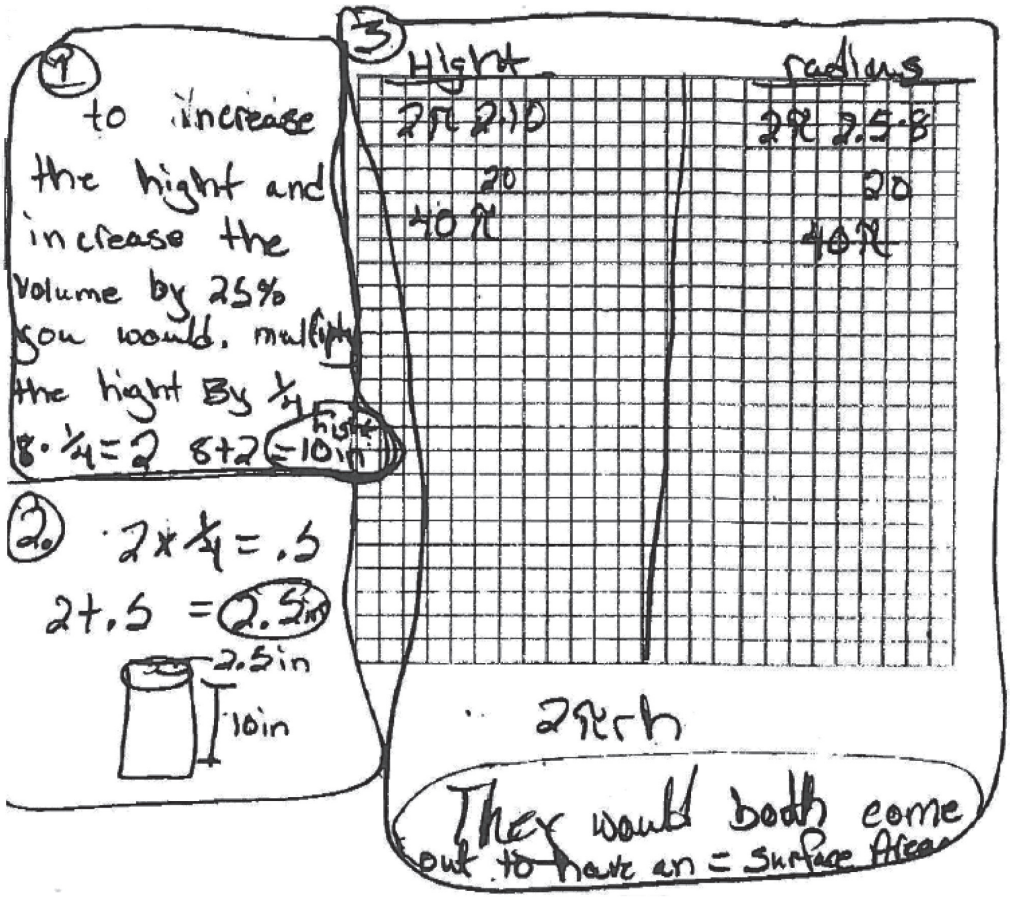
Part 1		**Points**
Correct height correct & complete procedure:	10 (in) 8 + 2 = 10	**1½**

Part 2

Incorrect radius incorrect procedure:	2.25 2 + .5 = 2.5	–

Part 3

Correct area based on Parts 1 & 2 with tops and bottoms omitted:	equal surface areas $SA(h = 10) = 2\pi \cdot 2 \cdot 10 = 40\pi$ *(lateral surface area)* $SA(r = 2.5) = 2\pi \cdot 2.5 \cdot 8 = 40\pi$ *(lateral surface area)*	½
	Total Points	**2**



SCORE POINT: 1

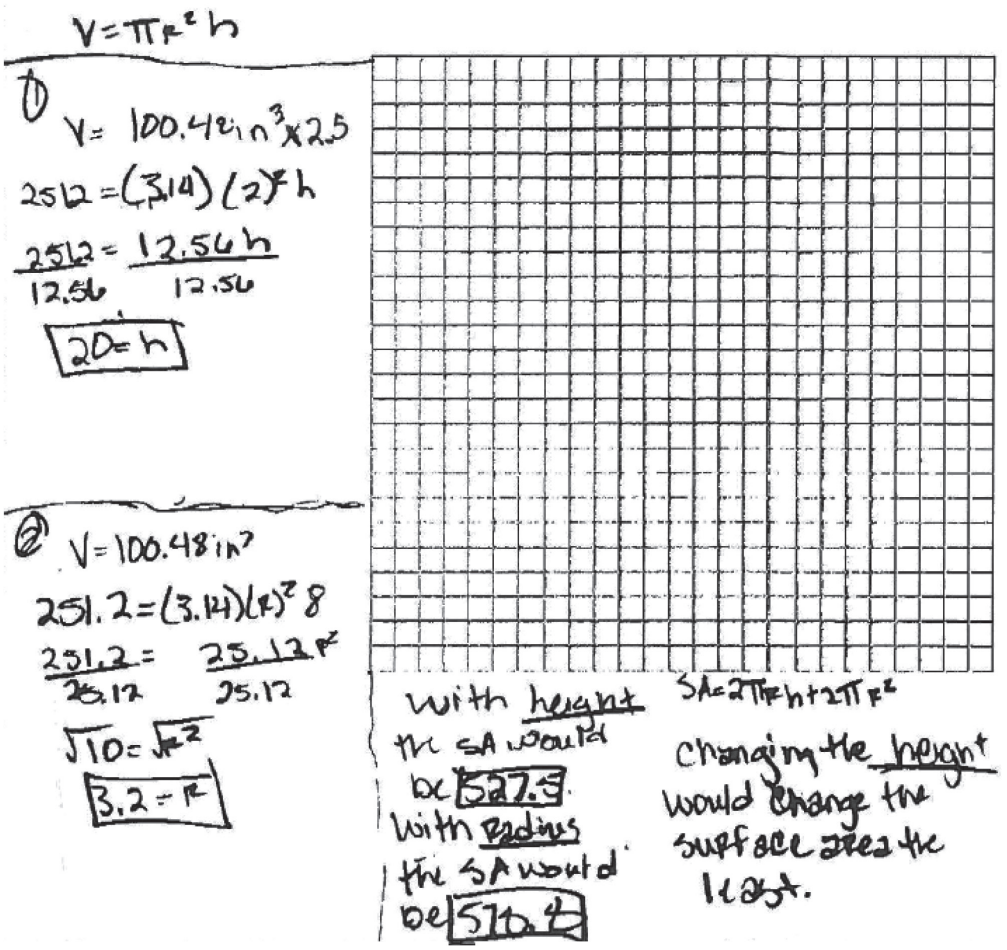
Part 1		**Points**
Incorrect height incorrect procedure:	20 V = 100.4 × 2.5 = 251.2	–

Part 2

Incorrect radius correct procedure based on Part 1 *V*:	3.2 *(not rounded to nearest 100th)* Sets up equation and solves to find r: 251.2 = (3.14)(r^2)8 etc.	**1**

Part 3

2 incorrect *SA*'s:	SA (*h* = 20) = 527.5 *(should be 276.5)* SA (*r* = 3.2) = 578.8 *(should be 225.2)*	–
	Total Points	**1**



SCORE POINT: 0

Part 1

		Points
Incorrect height incorrect procedure:	h = 41.87 Incorrect formula: $\frac{\pi r^2 h}{3}$	–

Part 2

Incorrect radius incorrect procedure:	65.41 Incorrect formula: $\frac{\pi r^2 h}{3}$	–

Part 3

Incorrect procedure:	"the one with the smaller measurements"	–
	Total Points	**0**

(1) h=41.87 in²

$\frac{3.14(2^2)8}{3}$ $\frac{3.14(2^2)10}{3}$

33.49 in² 41.87 in²

h=height

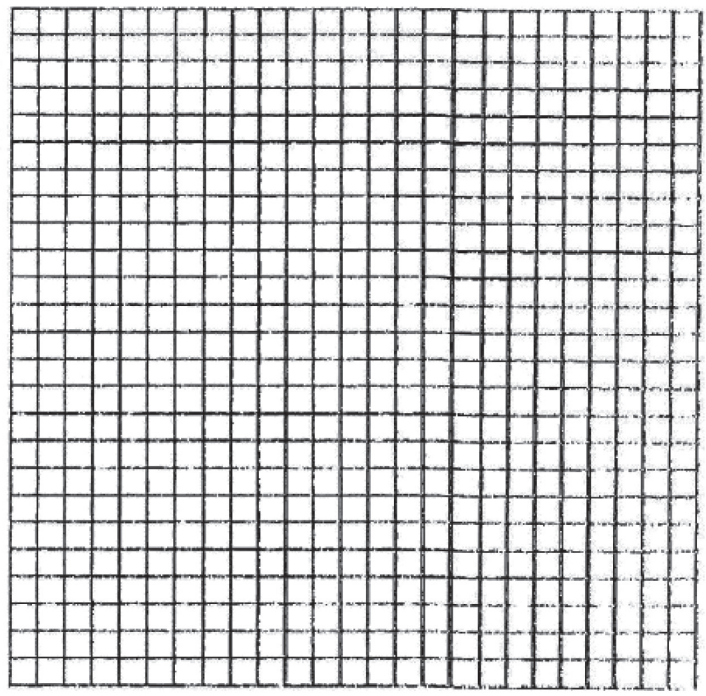
(2) r=65.41 in³

2.25% = 0.5

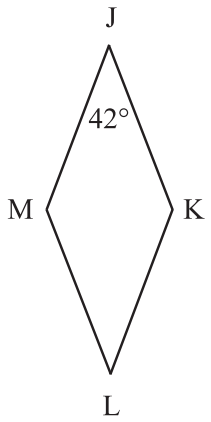
$\frac{3.14(0.5^2)10}{3}$ =

65.41

(3) the cylinder with the less surface area will be the one with the smaller measurements because with smaller units the surface area can not be very large.



D. Quadrilateral JKLM has four congruent sides.



1. What is the **most** specific name for quadrilateral JKLM? Explain your reasoning.

2. Determine m∠K. Show your work or explain how you found your answer.

3. MK = 7.6 cm and JL = 19.8 cm. What is the perimeter of JKLM? Round your answer to the nearest tenth of a centimeter. Show your work or explain how you found your answer.

BE SURE TO LABEL YOUR RESPONSES 1, 2, AND 3.

Item D Scoring Rubric—2011 Geometry

Score	Description
4	The student earns **4** points. The response contains no incorrect work. The correct label of "degrees" is included in Part 2.
3	The student earns **2½–3½** points and credit is earned in all 3 parts.
2	The student earns **2–2½** points and credit is earned in only 2 parts.
1	The student earns **½–1½** points or some minimal understanding is shown. Ex: Name in Part 1 is incorrect – Correct explanation to prove that the quadrilateral is a rhombus is included (with no other credit)
0	The student earns 0 points. No understanding is shown.
B	Blank—No Response. A score of "B" will be reported as "NA." (No attempt to answer the item. Score of "0" assigned for the item.)

SOLUTION AND SCORING

Note: Do not deduct for internal work that contains early-rounded or truncated values. Many students write these values for the sake of brevity, but use the exact value in the calculator to find their final answer.

4 points possible:

Part	Points
1	**1 point possible** 1 point: Correct name: **Rhombus** Correct explanation Give credit for the following or equivalent: Ex: "All four sides are congruent and the angles don't measure 90°, so it's a rhombus." Ex: "Rhombus - 4 sides are congruent but the angles aren't all congruent." Ex: "4 sides are the same length and we can tell from the measures of the angles that it's not a square, so it's a rhombus." Ex: "Rhombus because it has 2 sets *(or pairs)* of congruent angles." *(The fact that 4 sides are ≅ is given in the prompt, and the fact that the angles are not right angles is implied)* **OR** ½ point: • Correct name: **Rhombus** Work is incomplete, missing or incorrect
2	**1 point possible** 1 point: Correct $m\angle K$: **138** (degrees) Correct procedure is shown and/or explained Give credit for the following or equivalent: • *(Since JKLM is a rhombus, opposite angles are congruent and the sum of the measure of the angles is 360°.)* Ex: $2(42) + 2\,(m\angle K) = 360$ $2\,(m\angle K) = 276 \rightarrow m\angle K = 138$ • *(Since JKLM is a rhombus, adjacent angles are supplementary.)* Ex: $42 + m\angle K = 180$ $m\angle K = 138$ **OR** ½ point: • Measure is correct: **138** Work is incomplete or missing or • Measure is incorrect due to a calculation or copy error Correct procedure is shown and/or explained

Part	Points
3	**2 points possible** 2 points: Correct Perimeter: 42.4 (cm) Correct procedure is shown and/or explained Give credit for the following or equivalent: • Using the Pythagorean Theorem Ex: *(The diagonals of a rhombus are perpendicular bisectors. Therefore, the 4 triangles formed by the diagonals and the sides of the rhombus are congruent right triangles.)* Lengths of the legs of the triangles are: $\frac{7.6}{2} = 3.8 \quad \frac{19.8}{2} = 9.9$ *(not required)* $(3.8)^2 + (9.9)^2 = 14.44 + 98.01 = 112.45$ $\sqrt{112.45} = 10.604244... \rightarrow 10.6$ $4(10.6...) = 42.416977... \rightarrow 42.4$ Ex: $\sqrt{9.9^2 + 3.8^2} = 10.6$ $4(10.6) = 42.4$ • Using Trig. Functions Ex: $\sin 21° = \frac{3.8}{x}$ x = 10.6036... (or $\cos 69° = \frac{3.8}{x}$ etc.) $4(10.6) = 42.4$ Ex: $\sin 69° = \frac{9.9}{x}$ x = 10.6043... (or $\cos 21° = \frac{9.9}{x}$ etc.) $4(10.6) = 42.4$ **OR** 1½ points: • Perimeter is correctly rounded or truncated to a place other than the nearest 10th Correct procedure is shown and/or explained

Part	Points
3	**2 points possible (continued)** **OR** 1 point: • Perimeter is correct: 42.4 Work is incomplete or missing or • Perimeter is incorrect due to a calculation, copy or rounding error May or may not be rounded to the nearest 10th Correct procedure is shown and/or explained or • Perimeter is correctly rounded or truncated to a place other than the nearest 10th Work is incomplete (Ex: $10.6 \times 6 = 42.417$) or • The correct length of 1 side of the rhombus is found (10.6) Correct procedure is shown and/or explained Does not proceed to find Perimeter **OR** 1 point: • Perimeter is incorrect due to 2 calculation, copy or rounding errors May or may not be rounded to the nearest 10th Correct procedure is shown and/or explained

SCORE POINT: 4

Part 1		**Points**
Correct name with correct explanation:	Rhombus "4 ≅ sides ...does not have 4 right angles"	**1**

Part 2

Correct measure with correct procedure:	138 (degrees) $\frac{360-[2(42)]}{2} =$ etc.	**1**

Part 3

Correct perimeter with correct procedure:	42.4 (rounded to the nearest 10th) (cm) Uses the Pythagorean Theorem to find length of 1 side: $9.9^2 + 3.8^2 = c^2$ etc. $4(\sqrt{112.45}) = 42.4$	**2**
	Total Points	**4**

Correct label of "degrees" is included in Part 2

1.) Rhombus.

It has 4 ≅ sides and the opposite ∠s are ≅. It does not have four right angles so it cannot be named a square or rectangle.

2.) $\angle K \cong \angle M$
$\angle J \cong \angle L$

so → $\frac{360-[2(42)]}{2} = \angle K$

$\frac{360-[2(42)]}{2} = \angle K$

$\frac{360-84}{2} = \angle K$

$\frac{276}{2} = \boxed{138° = \angle K}$

3.) $19.8/2 = 9.9$
$7.6/2 = 3.8$

$a^2+b^2=c^2$

side → $9.9^2+3.8^2=c^2$

$98.01+14.44=c^2$

$112.45=c^2$

$\sqrt{112.45}$ = side

$4(\sqrt{112.45})$ = perimeter

$\boxed{42.4\text{ cm}}$ ≈ perimeter

SCORE POINT: 3

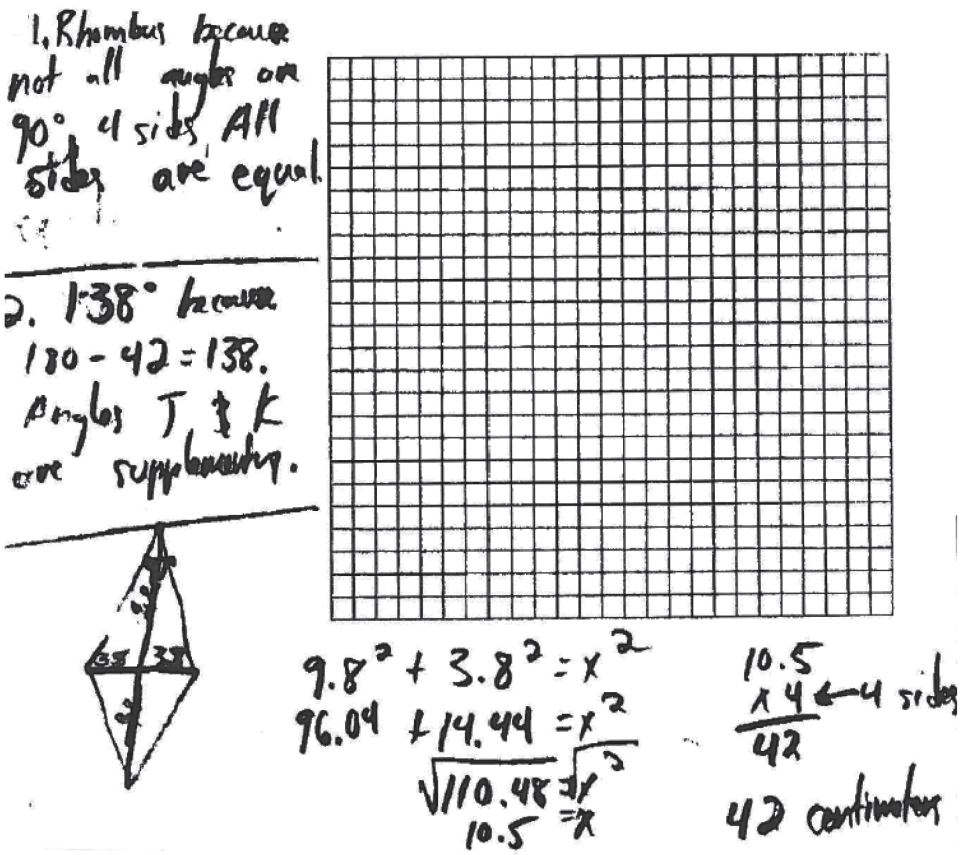
Part 1		**Points**
Correct name with correct explanation:	Rhombus "not all angles are 90° ...all sides are equal"	**1**

Part 2

Correct measure with correct procedure:	128 (degrees) 180 – 42 = 138	**1**

Part 3

Incorrect perimeter with correct procedure:	42 *(incorrect due to calc. er., not rounded to 10th's)* Uses the Pythagorean Theorem to find length of 1 side: $9.8^2 + 3.8^2 = x^2$ ... $10.5 = x$ $10.5 \cdot 4 = 42$ (9.8 *calculation error*)	**1**
	Total Points	**3**

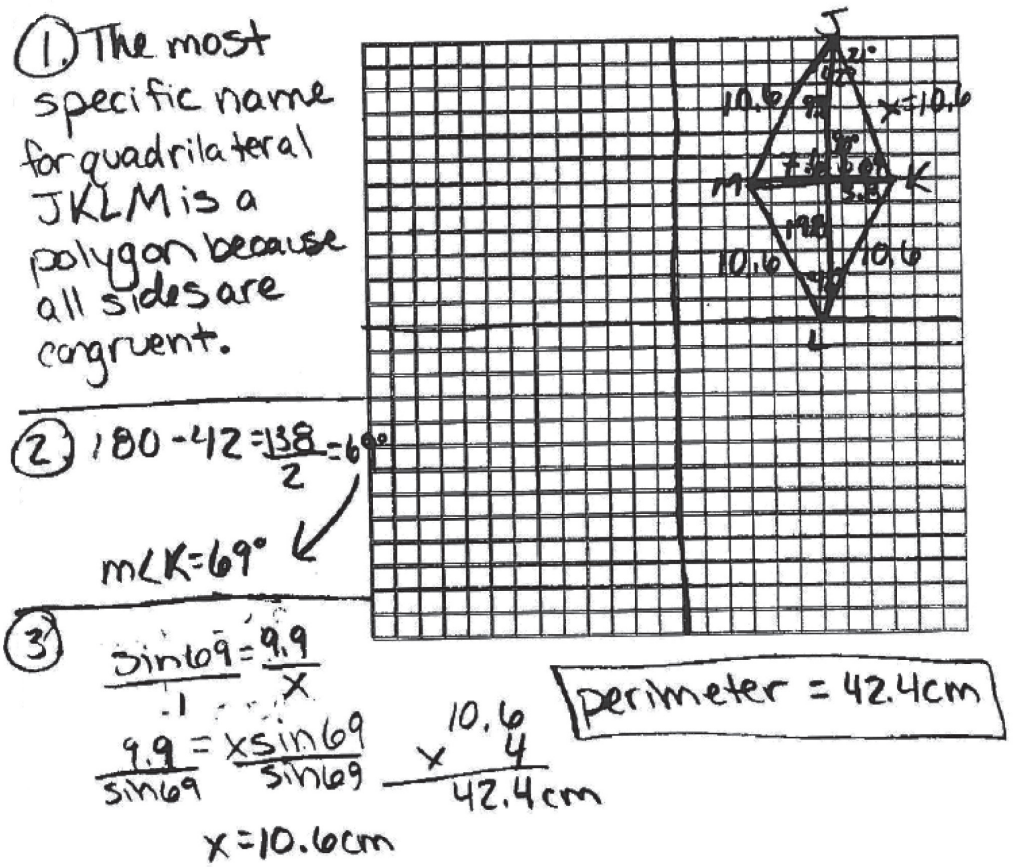


SCORE POINT: 2

Part 1		**Points**
Incorrect name:	Polygon *Note: No credit is given if the name is incorrect*	–

Part 2		
Incorrect measure with incorrect procedure:	69 (degrees) 180 – 42 = 138 138 ÷ 2 = 69	–

Part 3		
Correct perimeter with correct procedure:	42.4 (rounded to the nearest 10th) (cm) Uses Trigonometry to find the length of 1 side: $\sin 69 = \frac{9.9}{x}$ $x = 10.6$ $10.6 \cdot 4 = 42.4$	**2**
	Total Points	**2**

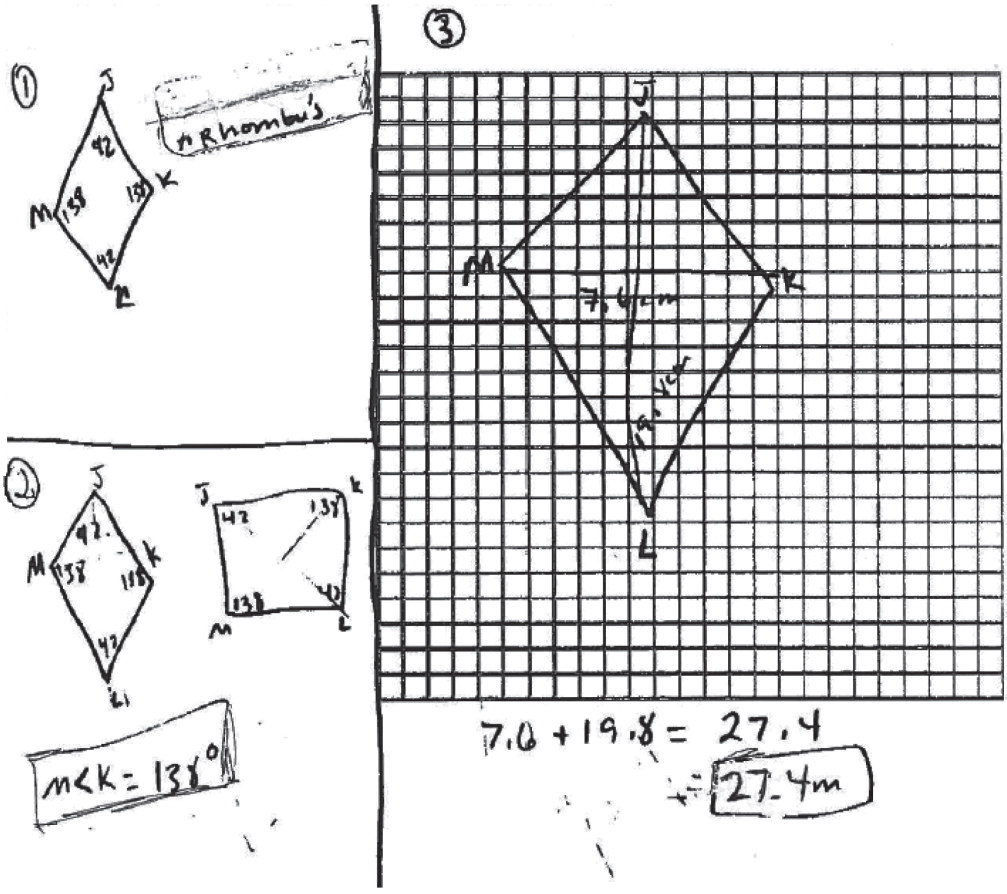


SCORE POINT: 1

Part 1		**Points**
Correct name with missing explanation:	Rhombus	½

Part 2		
Correct measure with missing procedure:	138 (degrees)	½

Part 3		
Incorrect perimeter with incorrect procedure:	27.4 7.6 + 19.8 = 27.4	–
	Total Points	**1**



SCORE POINT: 0

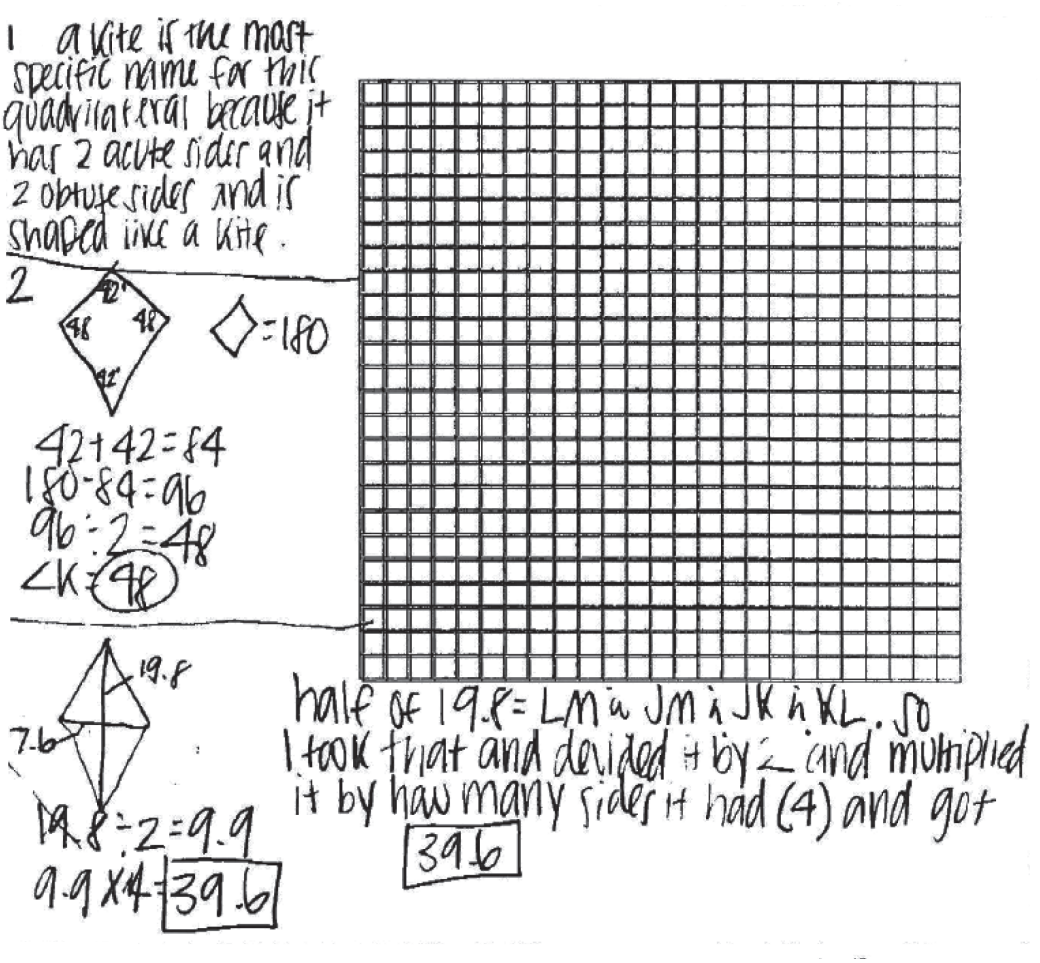
Part 1		**Points**
Incorrect name:	Kite *Note: No credit is given if the name is incorrect*	–

Part 2

Incorrect measure with incorrect procedure:	48 (degrees) 42 + 42 = 84 180 – 84 = 96 96 ÷ 2 = 48	–

Part 3

Incorrect perimeter with incorrect procedure:	39.6 19.8 ÷ 2 = 9.9 9.9 • 4 = 39.6	–
	Total Points	**0**

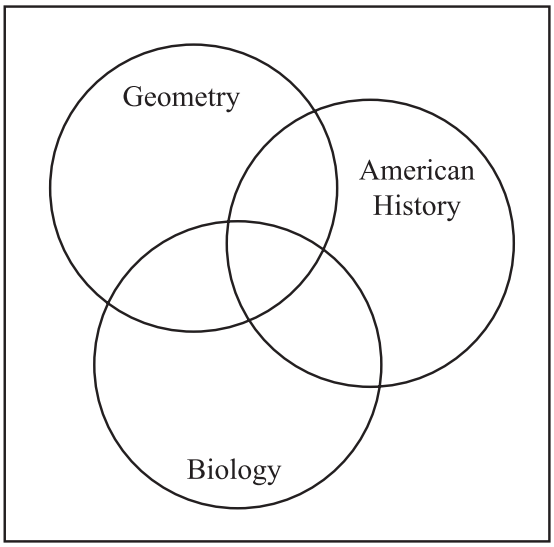


E. A middle school has the following information on its 75 seventh-grade students:

- 33 students are taking Biology
- 39 students are taking American History
- 43 students are taking Geometry
- 12 students take Biology and American History
- 21 students take Geometry and Biology
- 17 students take Geometry and American History
- 7 students are taking Biology, Geometry, and American History

Copy the Venn diagram below into your Student Answer Document.

Seventh Grade Students



1. Complete the Venn diagram based on the information given. Be sure to include those students who take none of these classes.

2. How many students are taking **fewer than** 2 of these courses? Show your work or explain how you found your answer.

3. How many students are taking **exactly** 2 of these courses? Show your work or explain how you found your answer.

BE SURE TO LABEL YOUR RESPONSES 1, 2, AND 3.

Item E Scoring Rubric—2011 Geometry

Score	Description
4	The student earns **5** points. Labels of "Geometry" or "G", "American History", "AH", or "H", and "Biology" or "B" (or equivalent) are included in Venn Diagram No incorrect work is included.
3	The student earns **3½–4½** points.
2	The student earns **3** points.
1	The student earns **½–1½** points or some minimal understanding is shown. Ex: Sum of #'s given in Venn Diagram is 75 (with no other credit)
0	The student earns **0** points. No understanding is shown.
B	Blank—No Response. A score of "B" will be reported as "NA." (No attempt to answer the item. Score of "0" assigned for the item.)

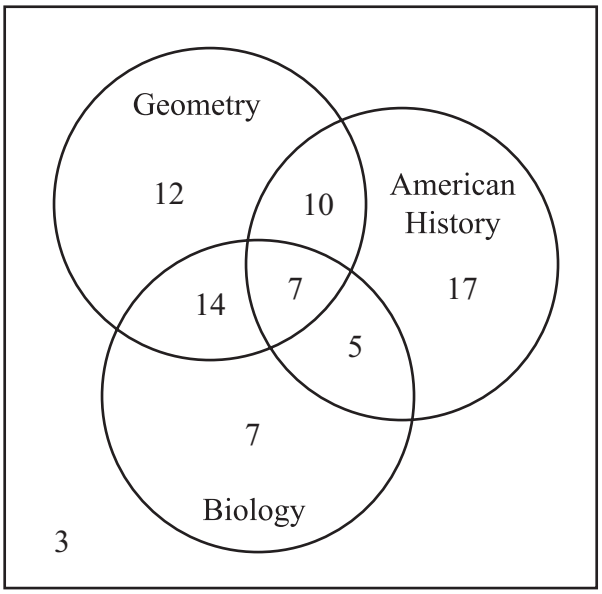
SOLUTION AND SCORING

5 points possible:

Part	Points
1	**3 points possible**

Ex:

Seventh Grade Students



Notes:

- **To receive credit, 3 intersecting circles must be drawn and positioned as in prompt. Labels of "Geometry" or "G", "American History", "AH", or "H", and "Biology" or "B" (or equivalent are required at the "4" level.**
- **Work is not required**
- **Credit is given, as indicated below, when correct values and no others are listed.**

3 points: **All 8 values** are correct and are placed in the appropriate positions, as shown in the diagram above.

OR

2½ points: **7 values within all 3 circles** are correct and are placed in the appropriate positions. "3" (# taking no courses) is incorrect or missing.

OR

2 point: All **values within 1 or 2 circles** are correct and are placed in the appropriate positions Other value(s) are incorrect or missing.

Ex: Geometry: 12, 10, 14 and 7 are correctly placed
Biology: 7, 14, 5 and 7 are correctly placed
Remaining #'(s) are incorrect or missing

Part	Points
1	**3 points possible (continued)** Ex: A. History: 17, 5, 10 and 7 are correctly placed Remaining #'(s) are incorrect or missing **OR** 1½ point: • All 4 values in the **intersections (2 or 3 courses)** of the 3 circles, **14, 10, 5 and 7** are correct and are placed in the appropriate positions. Values or representing 0 or 1 subject (3, 12, 17 and 7) are incorrect or missing. or • Values representing **"1 course only" (12, 17, 7)** and **"All 3 courses" (7)** are correct and are placed in the appropriate positions. Values representing "2 subjects only" (14, 7, 5) are incorrect or missing. **OR** 1 point: **Any 3 or 4 random values** are correct
2	**1 point possible** **Note: If no entry is made for # of students taking no courses, assume it is 0.** 1 point: Correct answer: **39** **(# taking fewer than 2 courses)** *(or correct # based on incorrect value(s) in Part 1)* Correct procedure is shown and/or explained Give credit for the following or equivalent: • 12 + 17 + 7 + 3 = 39 • 75 – (14 + 7 + 10 + 5) = 75 – 36 = 39 **OR** ½ point: • Answer is correct: 39 *(or correct # based on incorrect value(s) in Part 1)* Work is incomplete or missing or • Answer is incorrect due to a calculation or copy error Correct procedure is shown and/or explained or • Correct answer based on #'s taking exactly 1 course Number (≠0) taking no courses is ignored Correct procedure is shown and/or explained

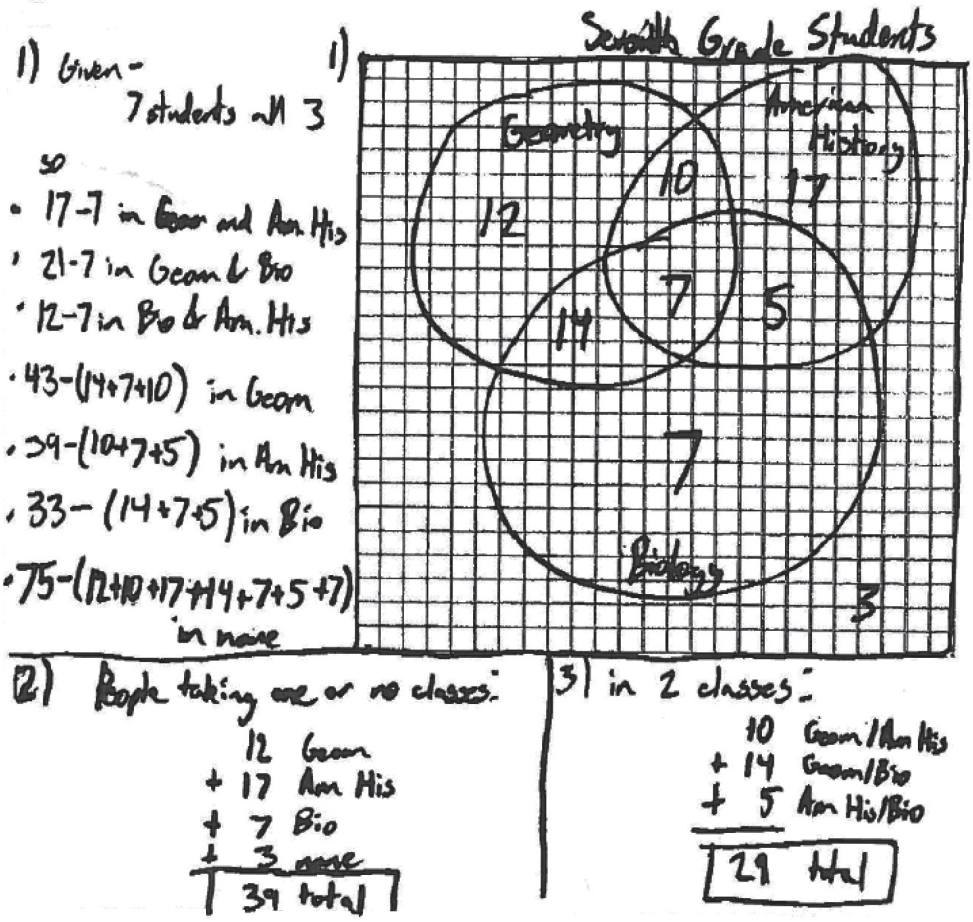
Part	Points
3	**1 point possible** 1 point: • Correct answer: **29** **(#taking exactly 2 courses)** *(or correct # based on incorrect value(s) in Part 1)* Correct procedure is shown and/or explained Give credit for the following or equivalent: • 10 + 14 + 5 = 29 • 75 – 39 – 7 = 29 **OR** ½ point: • Answer is correct: 29 *(or correct # based on incorrect value(s) in Part 1)* Work is incomplete or missing or • Answer is incorrect due to a calculation or copy error Correct procedure is shown and/or explained

SCORE POINT: 4

Part 1		**Points**
Correct diagram:	All 8 values are correct and are correctly positioned	**3**

Part 2		
Correct answer with correct procedure:	39 12 + 17 + 7 + 3 = 39	**1**

Part 3		
Correct answer with correct procedure:	29 10 + 14 + 5 = 29	**1**
	Total Points	**5**

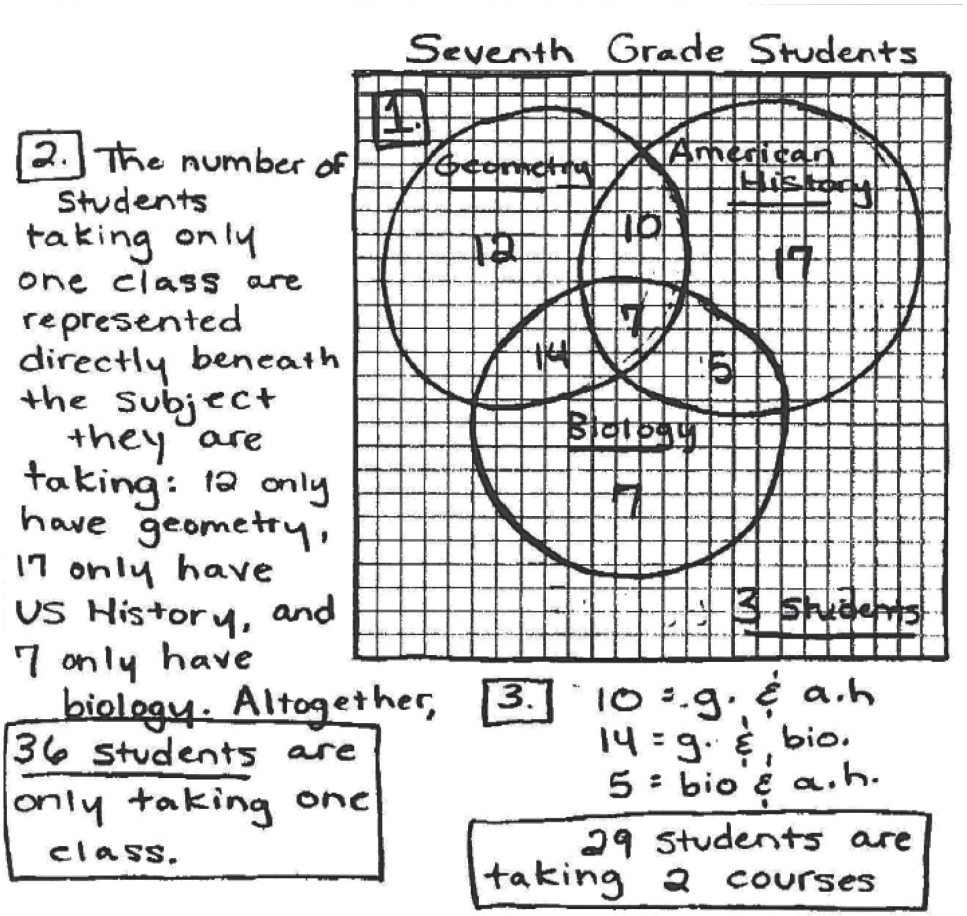


SCORE POINT: 3

Part 1		**Points**
Correct diagram:	All 8 values are correct and are correctly positioned	**3**

Part 2		
Correct answer/1 course correct procedure:	36 [# taking no courses (3) is ignored] 12 only geometry, 17 only US History, 7 only biology	½

Part 3		
Correct answer with correct procedure:	29 10 + 14 + 5 = 29 *(addition is implied)*	**1**
	Total Points	**4½**



SCORE POINT: 2

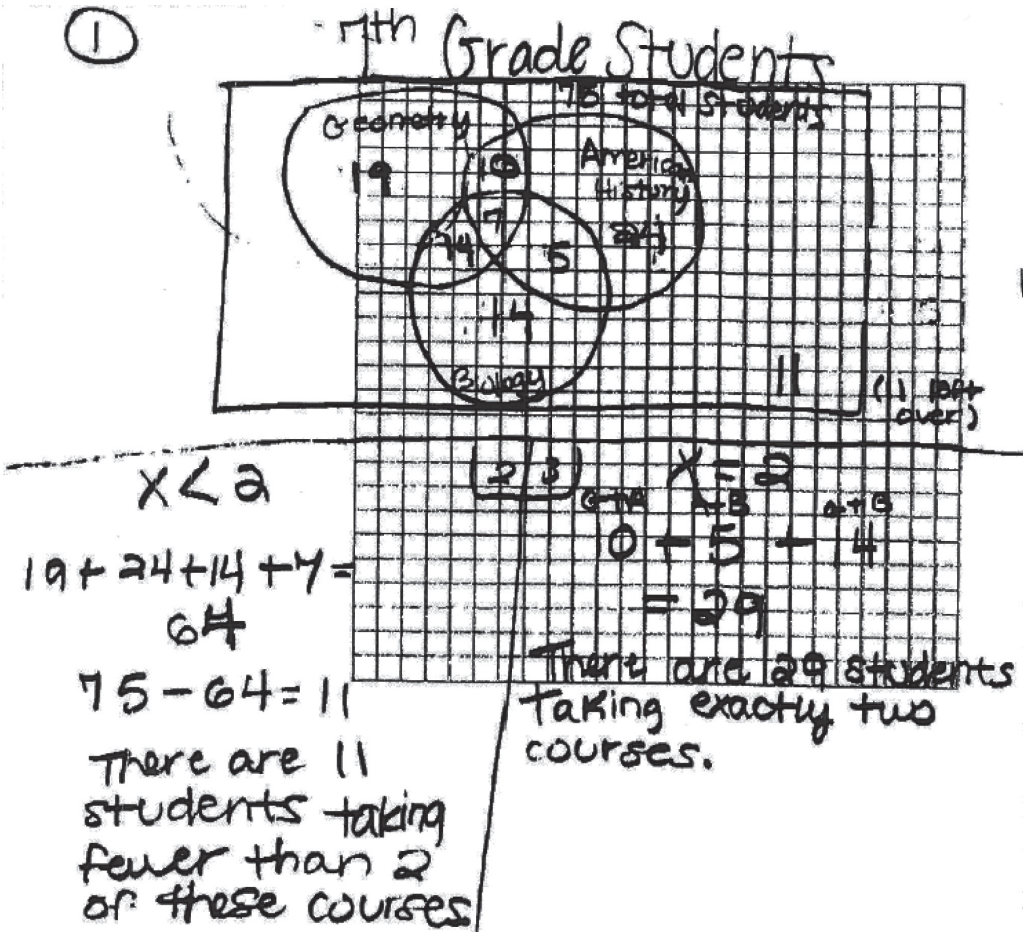
Part 1		**Points**
Partially correct diagram:	4 values in the intersection (2 or 3 courses) are correct Other values, (19, 24, 14, 11), are incorrect	**1½**

Part 2

Incorrect answer with incorrect procedure:	11 Subtracts sum of #'s taking only 1 course (based on Pt. 1) (19, 14, 24) and all 3 courses (7) from 75	–

Part 3

Correct answer with correct procedure:	29 10 + 5 + 14 = 29	**1**
	Total Points	**2½**



SCORE POINT: 1

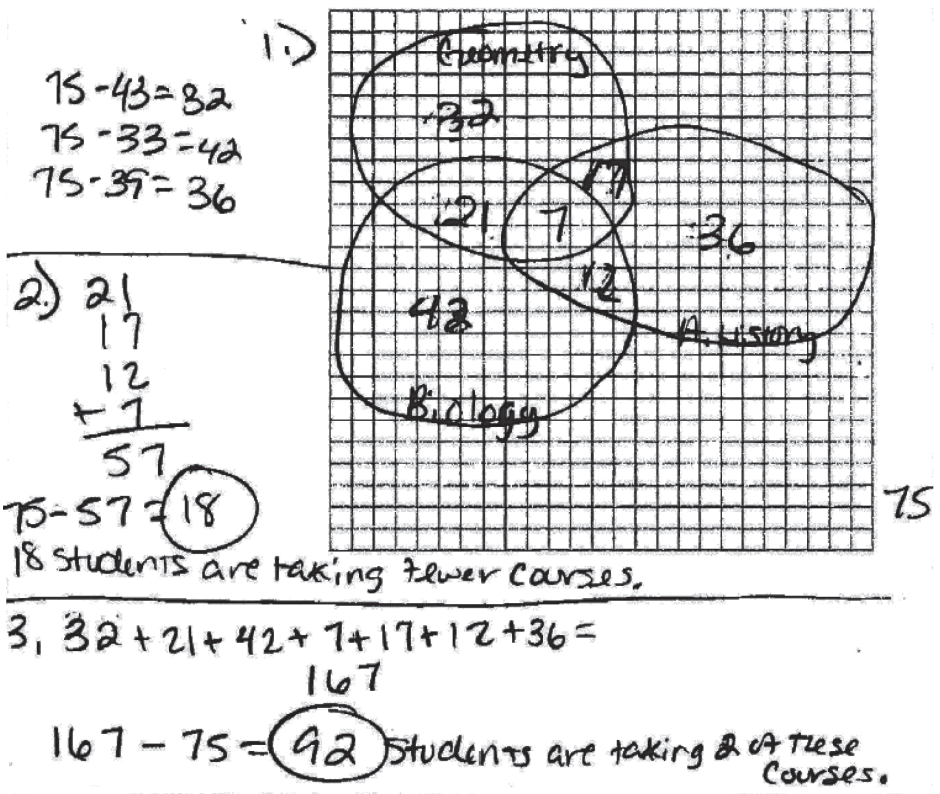
Part 1		**Points**
Insufficient diagram:	Only 1 value is correct (7) *(no credit)*	–

Part 2

Correct answer based on Part 1 with correct procedure:	18 *(correct based on Part 1 diagram and method used)* Subtracts # taking 2 or 3 courses from 75 (total # of students): 21 + 17 + 12 + 7 = 57 75 – 57 = 18 *Note: 32 + 36 + 42 = 110 would also be acceptable*	**1**

Part 3

Incorrect answer with incorrect procedure:	92 Subtracts total # of students from sum of #'s in diagram: 32 + 21 + 42 + 7 + 27 + 12 + 36 = 167 167 – 75 = 92	–
	Total Points	**1**

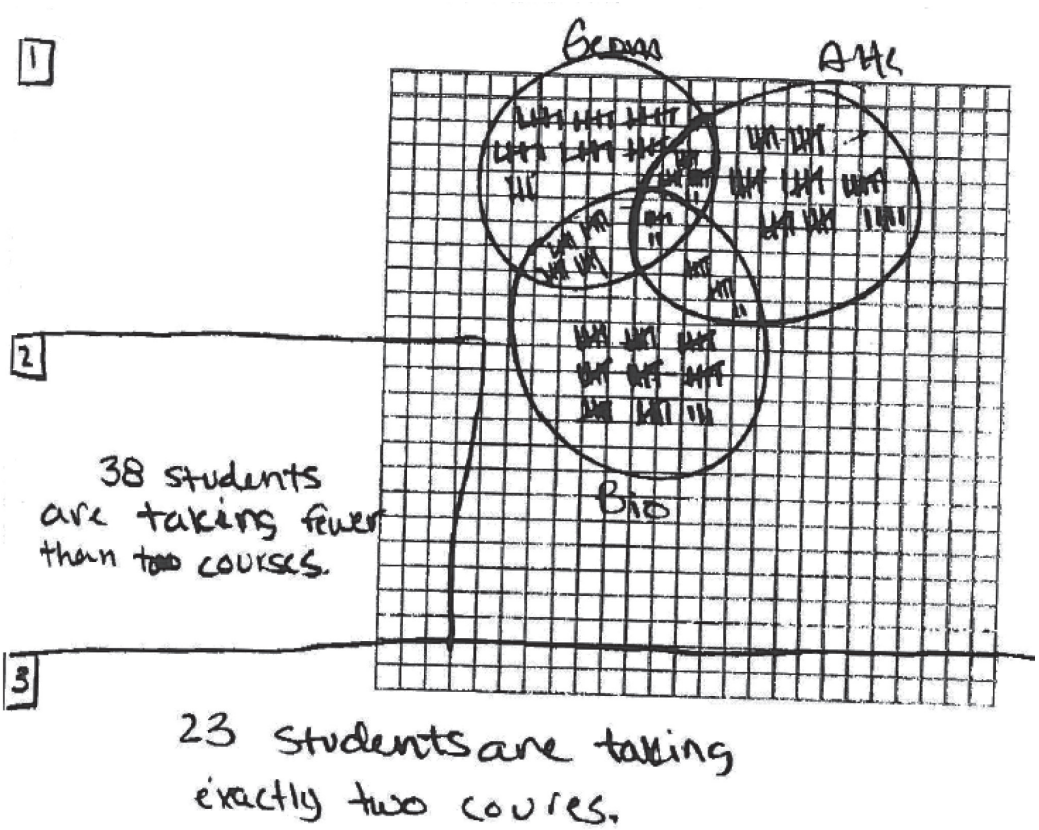


SCORE POINT: 0

Part 1		**Points**
Insufficient diagram:	Uses tally marks to represent #'s	–

Part 2		
Incorrect answer with missing procedure:	38	–

Part 3		
Incorrect answer with missing procedure:	23	–
	Total Points	**0**



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Arkansas Comprehensive Testing, Assessment, and Accountability Program

DEVELOPED FOR THE ARKANSAS DEPARTMENT OF EDUCATION, LITTLE ROCK, AR 72201

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