ACTAAP

Arkansas Comprehensive Testing, Assessment, and Accountability Program

Released Item Booklet

Geometry
End-of-Course Examination

April 2010 Administration

Arkansas Department of Education

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PART I Overview

The criterion-referenced tests implemented as part of the **Arkansas Comprehensive Testing, Assessment, and Accountability Program** (ACTAAP) are being developed in response to Arkansas Legislative Act 35, which requires the State Board of Education to develop a comprehensive testing program that includes assessment of the challenging academic content standards defined by the Arkansas Curriculum Frameworks.

As part of this program, students in Arkansas public schools who had completed or were completing Geometry by the end of the spring semester participated in the *Geometry End-of-Course Examination* in April 2010.

This Released Item Booklet for the *Geometry End-of-Course Examination* contains test questions or items that were asked of students during the April 2010 operational administration. The test items included in Part II of this booklet are some of the items that contributed to the student performance results for that administration.

Students were given approximately two hours each day to complete assigned test sessions during the two days of testing in April 2010. Students were permitted to use a calculator for both multiple-choice and open-response items. Students were also supplied with a reference sheet to be used so that all students would have equal access to this information during testing. (See the reference sheet on page 19 of this booklet.) All of the multiple-choice items within this booklet have the correct response marked with an asterisk (*).

The development of the *Geometry End-of-Course Examination* was based on the *Arkansas Geometry Mathematics Curriculum Framework*. This framework has distinct levels: Strands to be taught in concert, Content Standards within each Strand, and Student Learning Expectations within each Content Standard. An abridged version of the *Arkansas Geometry Mathematics Curriculum Framework* can be found in Part III of this booklet. It is important to note that this abridged version lists only the predominant Strand, Content Standard, and Student Learning Expectation associated with each item. However, since many key concepts within the *Arkansas Geometry Mathematics Curriculum Framework* are interrelated, in many cases there are other item correlations or associations across Strands, Content Standards, and Student Learning Expectations.

Part IV of the Released Item Booklet contains a tabular listing of the Strand, Content Standard, and Student Learning Expectation that each question was designed to assess. The multiple-choice and open-response items found on the *Geometry End-of-Course Examination* were developed in close association with the Arkansas education community. Arkansas teachers participated as members of the Geometry Content Advisory Committee, providing routine feedback and recommendations for all items. The number of items associated with specific Strands, Content Standards, and Student Learning Expectations was based on approximate proportions suggested by the Content Advisory Committee, and their recommendations were accommodated to the greatest extent possible given the overall test design. Part IV of the Released Item Booklet provides Arkansas educators with specific information on how the *Geometry End-of-Course Examination* items align or correlate with the *Arkansas Geometry Mathematics Curriculum Framework* to provide models for classroom instruction.

PART I Scoring Student Responses to Geometry Open-Response Items

While multiple-choice items are scored by machine to determine if the student chose the correct answer from four options, responses to open-response items must be scored by trained "readers" using a pre-established set of scoring criteria.

The Arkansas Geometry Rangefinding Committee assisted in the development of the scoring criteria. The committee comprises active Arkansas educators with expertise in mathematics education.

Reader Training

Before readers are allowed to begin assigning scores to any student responses, they go through intensive training. The first step in that training is for the readers to read the Geometry open-response items as they appear in the test booklet and to respond—just as the student test takers are required to do. This step gives the readers some insight into how the students might have responded. The next step is the readers' introduction to the scoring rubric. All of the specific requirements of the rubric are explained by the Scoring Director who has been specifically trained to lead the scoring group. Then responses (anchor papers) that illustrate the score points of the rubric are presented to the readers and discussed. The goal of this discussion is for the readers to understand why a particular response (or type of response) receives a particular score. After discussion of the rubric and anchor papers, readers practice scoring sets of responses that have been pre-scored and selected for use as training papers. Detailed discussion of the responses and the scores they receive follows.

After three or four of these practice sets, readers are given "qualifying rounds." These are additional sets of pre-scored papers, and, in order to qualify, each reader must score in exact agreement on at least 80% of the responses and have no more than 5% non-adjacent agreement on the responses. Readers who do not score within the required rate of agreement are not allowed to score the *Geometry End-of-Course Examination* responses.

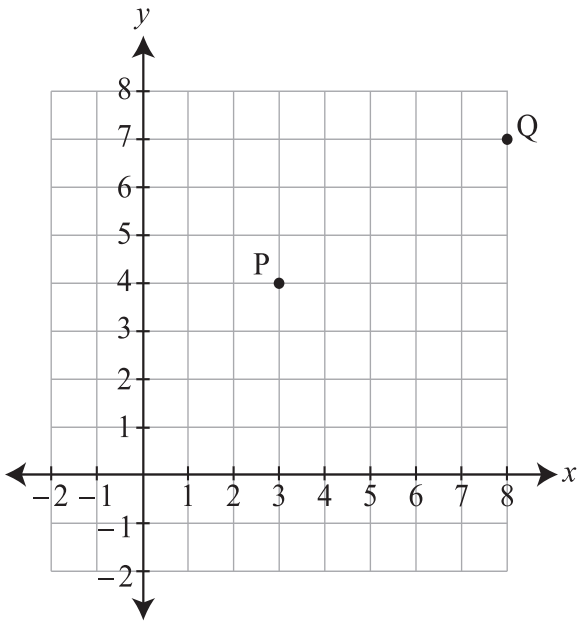
Once scoring of the actual student responses begins, readers are monitored constantly throughout the project to ensure that they are scoring according to the criteria. Daily and cumulative statistics are posted and analyzed, and Scoring Directors or Team Leaders reread selected responses scored by the readers. These procedures promote reliable and consistent scoring. Any reader who does not maintain an acceptable level of agreement is dismissed from the project.

Scoring Procedures

All student responses to the *Geometry End-of-Course Examination* open-response test items are scored independently by two readers. Those two scores are compared, and responses that receive scores that are non-adjacent (a "1" and a "3," for example) are scored a third time by a Team Leader or the Scoring Director for resolution.

PART II Released Geometry Items

1. What is the distance from P to Q? Round your answer to the nearest tenth.



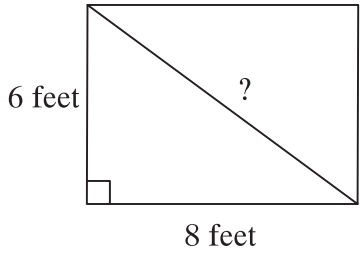
A. 3.0 units

B. 3.4 units

C. 5.0 units

*D. 5.8 units

Use the figure below to answer question 2.



(Not drawn to scale.)

2. Patty is building a 6 feet by 8 feet rectangular platform as shown above. All corners form 90° angles. What is the length of the diagonal of the platform?

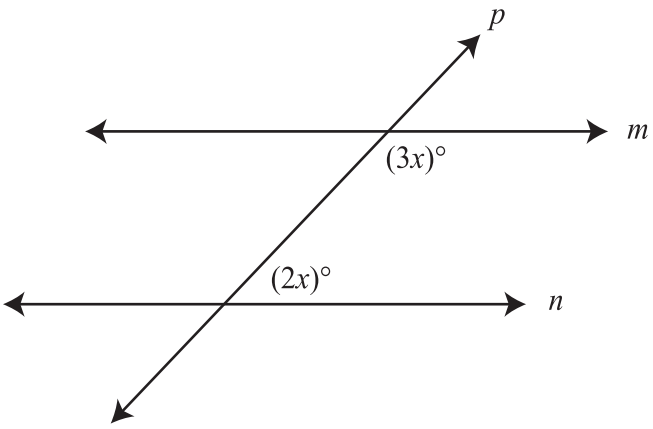
A. $\sqrt{14}$ feet

B. $\sqrt{28}$ feet

*C. 10 feet

D. 14 feet

3. In the figure below, $m \parallel n$.



What is the value of x?

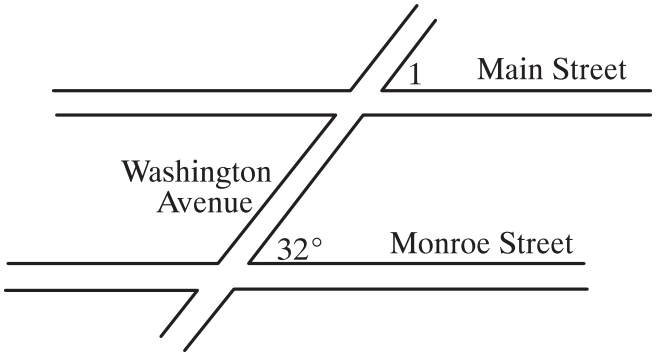
*A. 36

B. 72

C. 108

D. 180

Use the figure below to answer question 4.

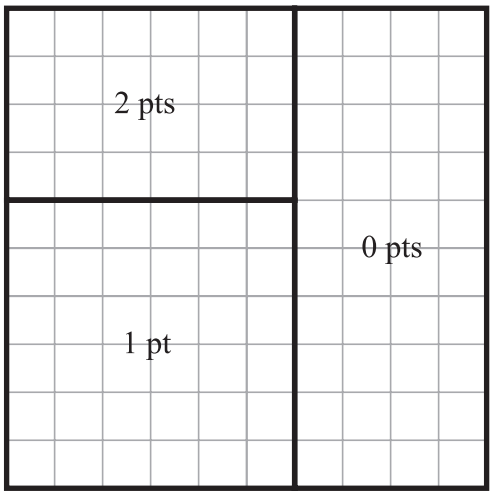


(Not drawn to scale.)

4. Main Street and Monroe Street are parallel. Washington Avenue will be built as a straight street intersecting both streets. It will form a 32° angle with Monroe Street shown in the figure above. What will be the measure of $\angle 1$?

*A. 32°
B. 58°
C. 122°
D. 158°

5. Simone and Josh designed the target below for the water balloon toss at the school carnival. Assuming the water balloon hits the target, what is the probability that it will hit the 2-point area?



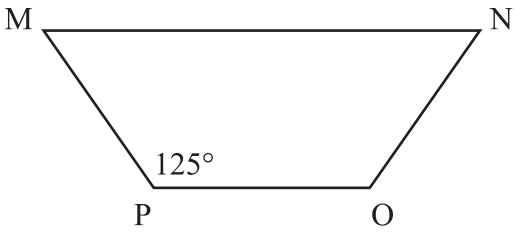
A. $\frac{2}{5}$

B. $\frac{3}{5}$

*C. $\frac{6}{25}$

D. $\frac{9}{25}$

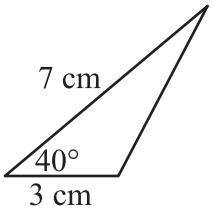
6. Quadrilateral MNOP is an isosceles trapezoid.



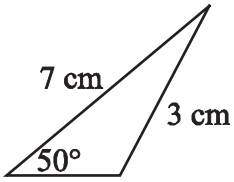
What is m∠N?

A. 35°
*B. 55°
C. 125°
D. 145°

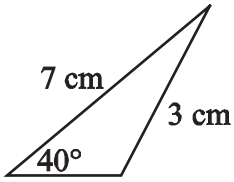
7. Which triangle must be similar to the triangle shown below?



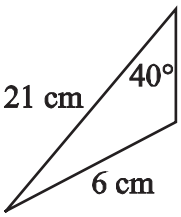
A.



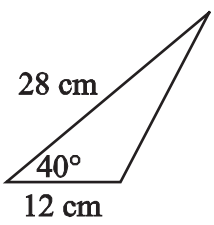
B.



C.



*D.



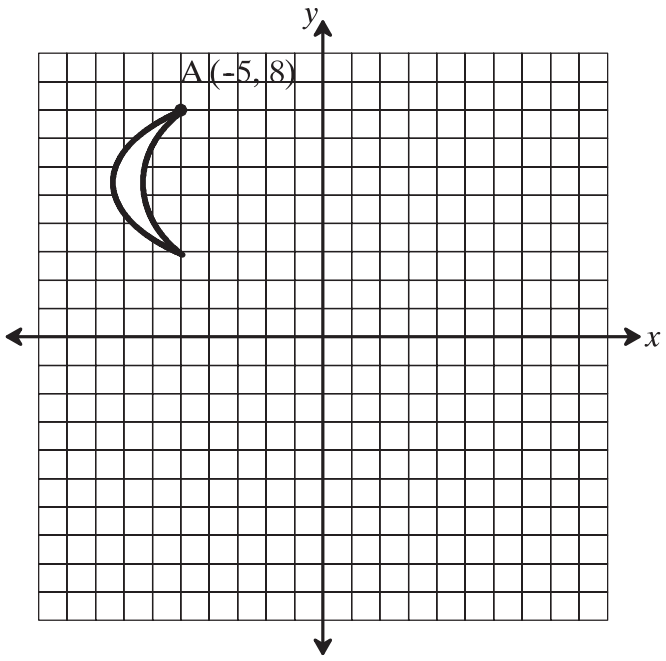
8. Theo, Amy, Lance, and Kaitlin live in four different states: Montana, Colorado, Virginia, and Arizona.

- Theo lives in a state that is spelled with an "i."
- Amy lives in a state that is spelled with an even number of letters.
- Lance does not live in a state that is spelled with more than one "a."

In which state does Kaitlin live?

*A. Montana
B. Colorado
C. Virginia
D. Arizona

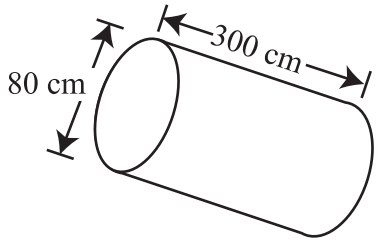
Use the graph below to answer question 9.



9. Josh is designing a cover for a paperback book. He is going to use the graphic shown above. He plans to reflect the graphic over the *y*-axis. What will be the coordinates of the reflection of point A?

A. (–5,–8)
*B. (5, 8)
C. (5,–8)
D. (8, 5)

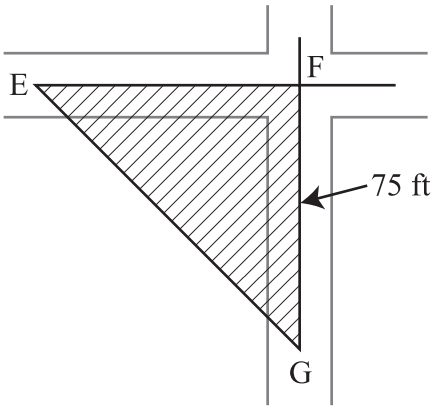
10. A cylindrical satellite, shown below, will be covered entirely in gold foil to reflect sunlight.



Approximately how many square centimeters of foil will be required to cover the entire surface area of the satellite?

A. 10053 cm^2
B. 75398 cm^2
C. 80425 cm^2
*D. 85451 cm^2

11. The city's zoning department has regulations to determine clear vehicle sight lines—called a Clear View Triangle—at all its intersections, as shown below.



If $m\angle F = 90°$, $m\angle G = 45°$, and FG = 75 feet, what is EG?

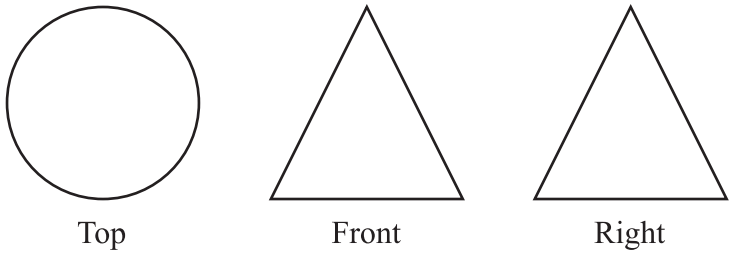
A. $\frac{75}{\sqrt{3}}$ ft

B. $\frac{75}{\sqrt{2}}$ ft

*C. $75\sqrt{2}$ ft

D. $75\sqrt{3}$ ft

12. The top, front, and right views of a three-dimensional figure are shown below.



What is the name for this figure?

*A. cone
B. sphere
C. cylinder
D. triangular pyramid

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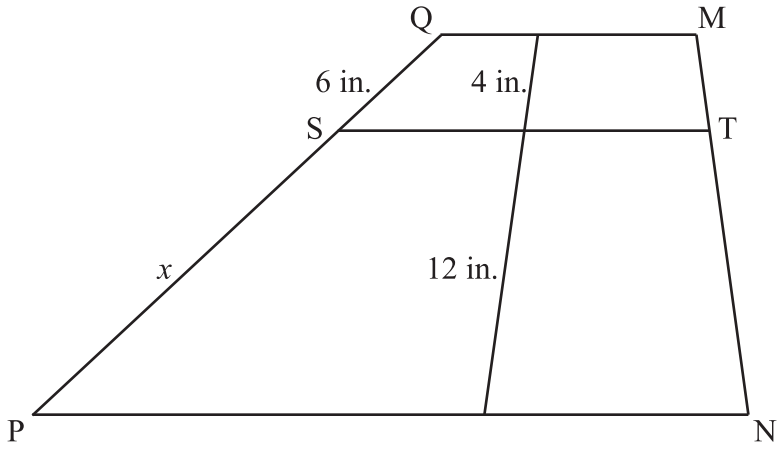
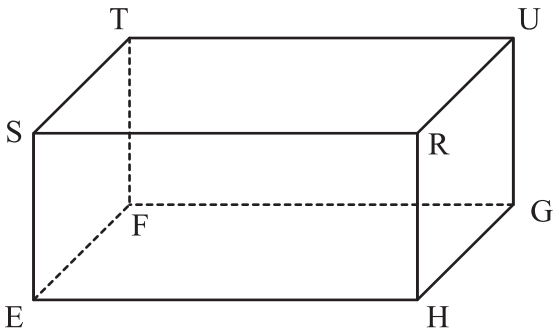
13. In the figure below, $\overline{QM} \parallel \overline{ST} \parallel \overline{PN}$.



What is the value of x?

A. 2 in.
B. 8 in.
C. 10 in.
*D. 18 in.

14. In the figure below, which pair of points is coplanar with $\overline{TR}$?



A. points G and H

*B. points F and H

C. points H and U

D. points H and S

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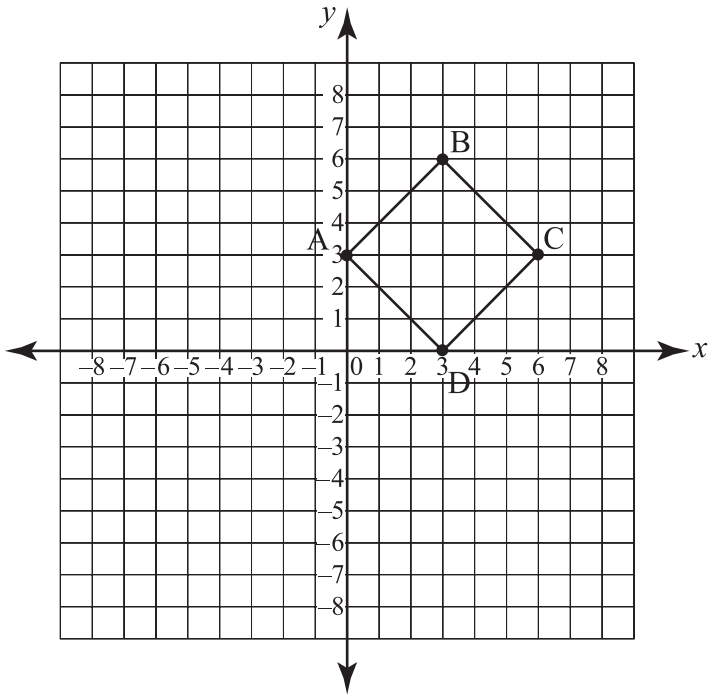
15. What are the equations of the lines that form the diagonals of the rhombus shown below?



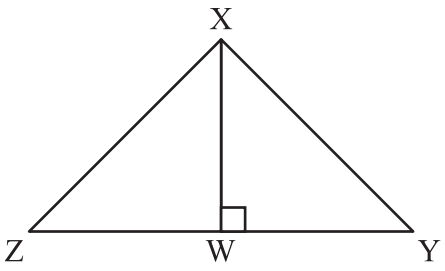
A. $y = -x + 3,\ y = -x + 9$

B. $x = 3,\ y = -3$

*C. $x = 3,\ y = 3$

D. $y = x + 3,\ y = x - 3$

16. In the figure below, $\overline{XW}$ bisects $\overline{YZ}$.



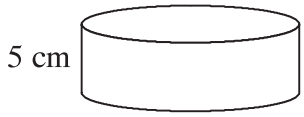
Which statement must be true?

A. $\overline{XW} \cong \overline{XY}$
B. $\overline{XW} \cong \overline{YW}$
C. $\angle ZXW \cong \angle XYW$
*D. $\angle ZXW \cong \angle YXW$

17. The equation of a circle is $(x-5)^2 + (y-7)^2 = 9$.
What is the radius of the circle?

*A. 3
B. 5
C. 7
D. 9

Use the figure below to answer question 18.



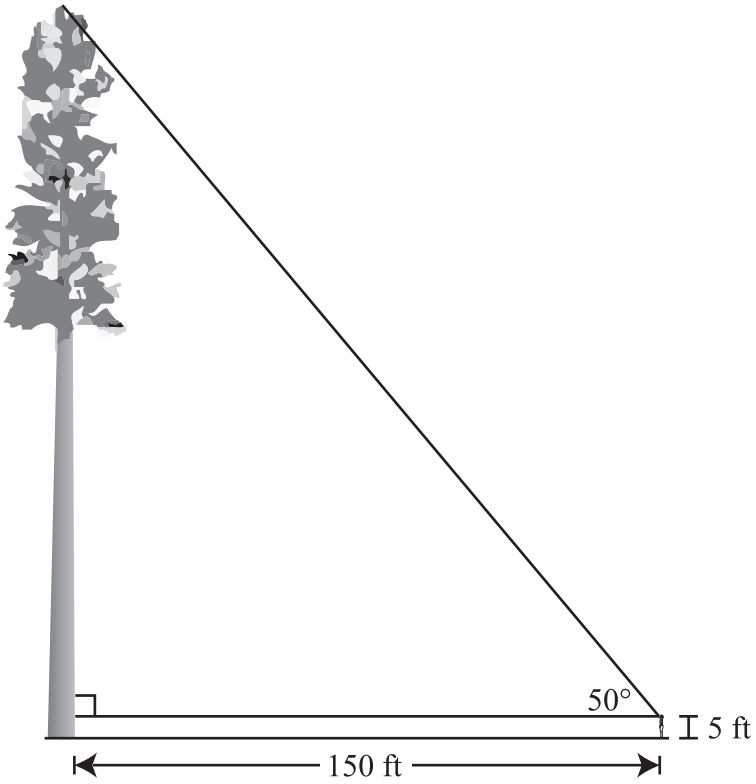
Diameter = 10 cm

(Not drawn to scale.)

18. A company doubles the height of the can shown above. The radius is made smaller so that the volume stays the same. What is the radius, to the nearest hundredth cm, of the new can? Use $\pi = 3.14$.

*A. 3.54 cm
B. 6.25 cm
C. 7.07 cm
D. 12.50 cm

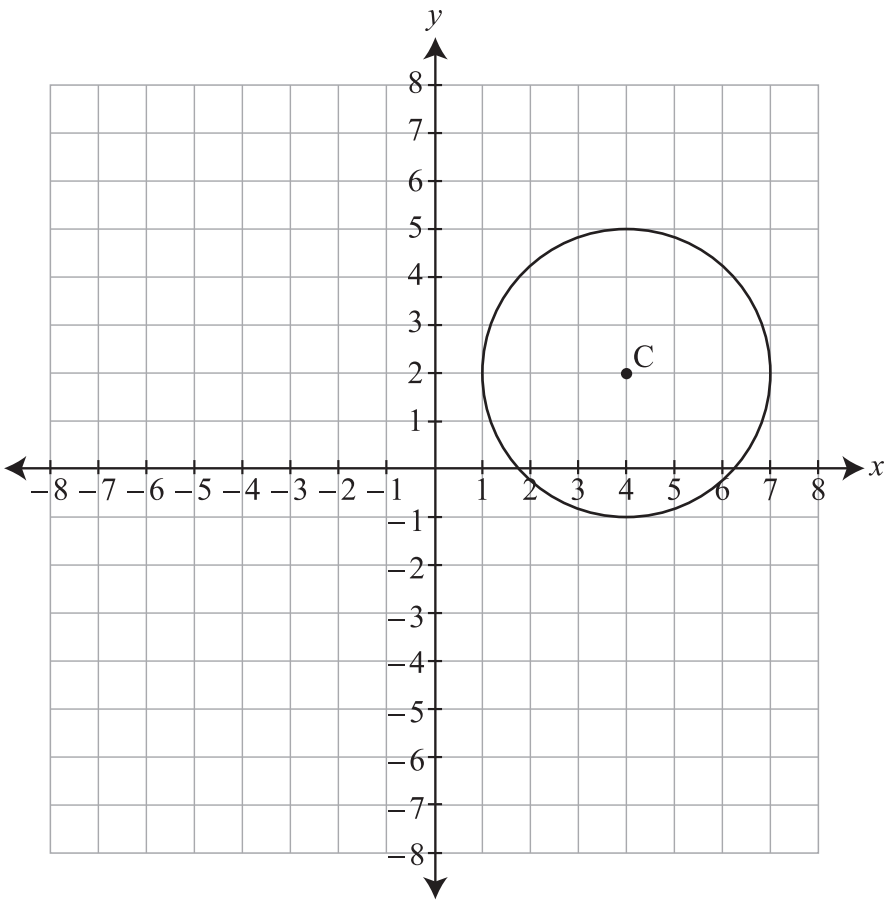
19. Roberto is standing 150 feet away from a pine tree, as shown below. The angle of elevation of his line of sight to the top of the tree is 50°. Roberto's eyes are 5 feet above the ground.



How tall is the tree? Round your answer to the nearest whole number.

A. 101 feet
B. 119 feet
C. 131 feet
*D. 184 feet

20. Demetria graphs circle C, shown below.



Next, Demetria plans to reflect circle C across the *y*-axis, then translate the image down by 3 units. What will be the new coordinates of the center of circle C?

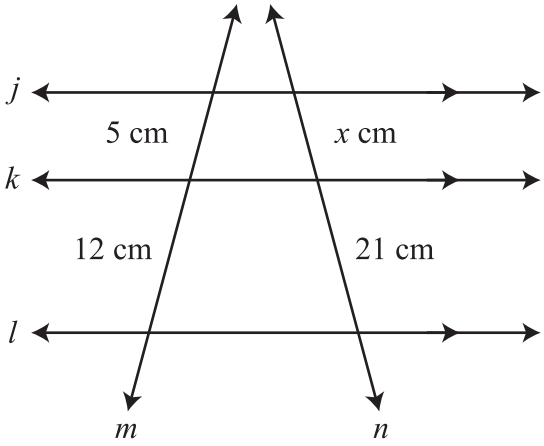
*A. (−4, −1)

B. (−4, 2)

C. (4, −5)

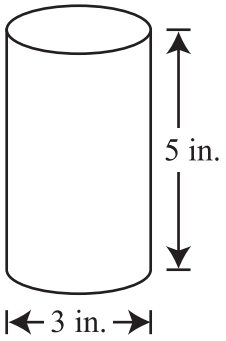
D. (4, −1)

21. What is the value of x in the figure below?



A. 7.86
*B. 8.75
C. 12.50
D. 14.00

22. Oscar is filling the cylindrical glass shown below with lemonade.

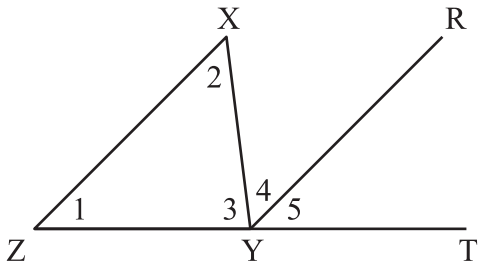


What is the maximum volume of lemonade the glass will hold? Round your answer to the nearest whole number.

A. 15 $in.^3$
*B. 35 $in.^3$
C. 47 $in.^3$
D. 61 $in.^3$

23. Given: ΔXYZ and $\overline{XZ} \parallel \overline{YR}$

Prove: $m\angle 1 + m\angle 2 + m\angle 3 = 180°$



Statement	Reason
1. $\overline{XZ} \parallel \overline{YR}$	1. Given
2. $\angle 1 \cong \angle 5$	2. ?
3. $\angle 2 \cong \angle 4$	3. Alternate interior angles of parallel lines are congruent.
4. $m\angle 3 + m\angle 4 + m\angle 5 = 180°$	4. $\angle ZYT$ is a straight angle.
5. $m\angle 1 + m\angle 2 + m\angle 3 = 180°$	5. Substitution

What is the reason in Step 2?

A. Vertical angles are congruent.
B. Complementary angles are congruent.
*C. Corresponding angles of parallel lines are congruent.
D. Alternate interior angles of parallel lines are congruent.

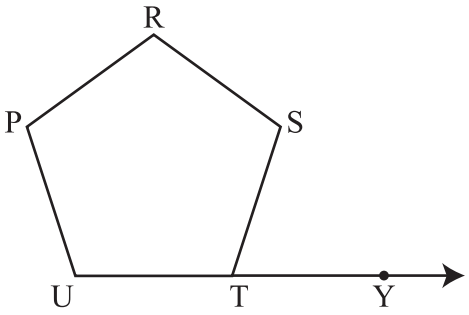
24. An object is cut to show a circular cross section. Which could **not** be the shape of the original object?

A. cone
*B. prism
C. sphere
D. cylinder

25. Which set of side lengths could represent the sides of a triangle?

A. 3 cm, 5 cm, 9 cm
B. 4 cm, 7 cm, 12 cm
*C. 5 cm, 9 cm, 13 cm
D. 6 cm, 11 cm, 18 cm

26. In the figure below, PRSTU is a regular pentagon.



What is m∠STY?

A. 36°
*B. 72°
C. 90°
D. 108°

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27. Given the pattern of identical squares below, how many squares would appear in Figure 5?

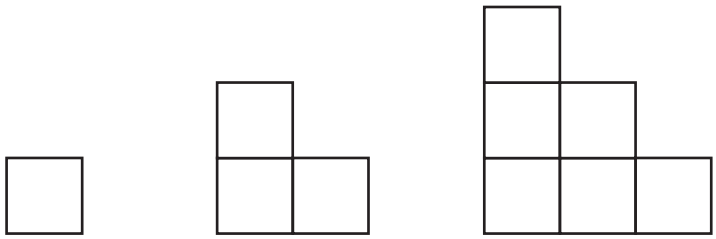
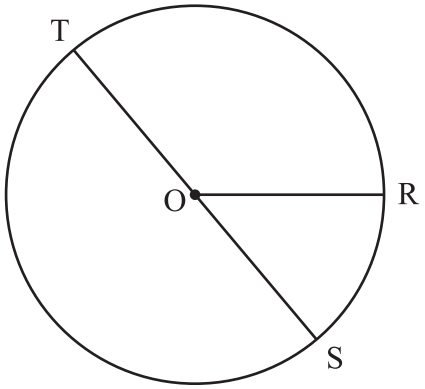


Figure 1 Figure 2 Figure 3

A. 10
B. 12
*C. 15
D. 20

28. In the figure below, $\overline{TS}$ is a diameter of circle O, and $m\overset{\frown}{RS} = 50°$.



What is m∠TOR?

A. 50°
B. 65°
C. 100°
*D. 130°

29. Which is an equation of the line perpendicular to the line with equation $y = \frac{3}{2}x - 2$ and passing through point (1, 5)?

A. $y = \frac{3}{2}x + \frac{7}{2}$

B. $y = \frac{2}{3}x + \frac{13}{3}$

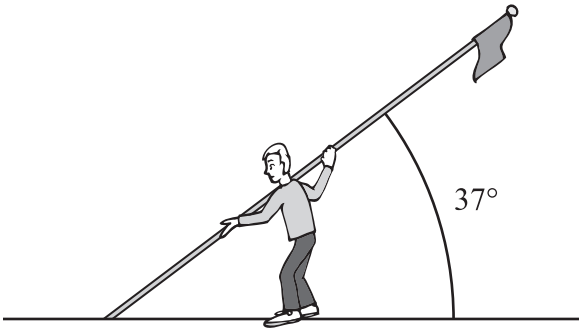
*C. $y = -\frac{2}{3}x + \frac{17}{3}$

D. $y = -\frac{3}{2}x + \frac{13}{2}$

30. In spherical geometry, a line is a great circle of a sphere. At how many points do any two lines in spherical geometry intersect?

A. 1
*B. 2
C. 3
D. 4

31. Jim is raising a flagpole. The angle between the pole and the ground is currently 37°.



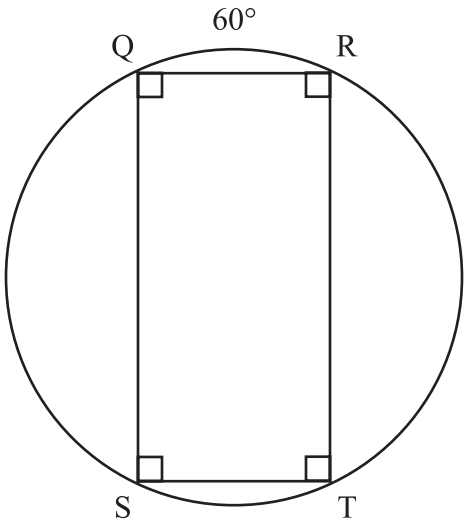
How many more degrees must Jim rotate the flagpole to make it perpendicular to the ground?

A. 37°
*B. 53°
C. 90°
D. 143°

32. A 3-inch by 5-inch photograph is placed on a copier and enlarged. The enlarged copy measures 11 inches on its longest side. What is the measurement of the shorter side of the enlarged copy?

*A. 6.6 inches
B. 8.5 inches
C. 9.0 inches
D. 12.8 inches

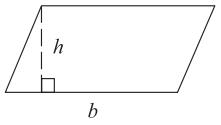
33. What is m$\overset{\frown}{RT}$ in the figure below?



A. 30°
B. 60°
C. 90°
*D. 120°

End-of-Course Mathematics Reference Sheet

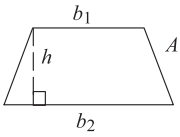
Parallelogram



P = sum of all sides

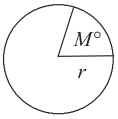
$A = bh$

Trapezoid



$A = \frac{h(b_1 + b_2)}{2}$

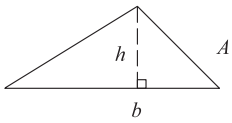
Arc and Sector



Arc Length = $\left(\frac{M}{360}\right) \times 2\pi r$

Sector area = $\left(\frac{M}{360}\right) \times \pi r^2$

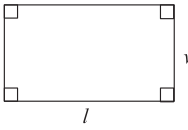
Triangle



P = sum of all sides

$A = \frac{bh}{2}$

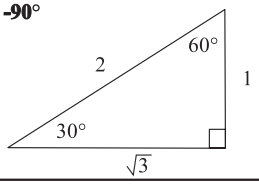
Rectangle



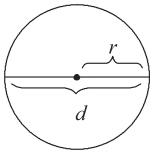
$P = 2l + 2w$

$A = l\,w$

30° -60° -90°



Circle



$C = 2\pi r$

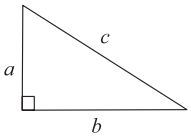
$C = \pi d$

$A = \pi r^2$

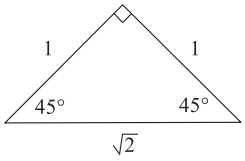
$\pi \approx 3.14$

Pythagorean Theorem

$a^2 + b^2 = c^2$

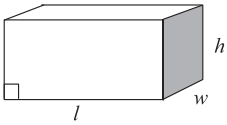


45° -45° -90°



Rectangular Solid

Volume = lwh

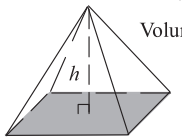


Surface area = $2lw + 2lh + 2wh$

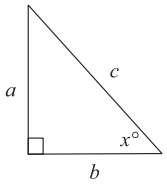
Pyramid

B = *area of base (shaded)*

Volume = $\frac{Bh}{3}$



Trigonometric Ratios

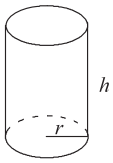


$\sin x° = \frac{a}{c}$

$\cos x° = \frac{b}{c}$

$\tan x° = \frac{a}{b}$

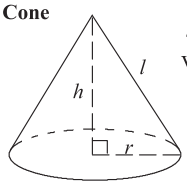
Cylinder



Volume = $\pi r^2 h$

Surface area = $2\pi rh + 2\pi r^2$

Cone

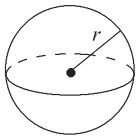


l = slant height

Volume = $\frac{\pi r^2 h}{3}$

Surface area = $\pi rl + \pi r^2$

Sphere



Volume = $\frac{4\pi r^3}{3}$

Surface area = $4\pi r^2$

Miscellaneous Formulas

Area of an equilateral triangle	$A = \frac{s^2\sqrt{3}}{4}$ s = length of a side
Distance	rate × time
Interest	principal × rate × time in years
Sum of the angles of a polygon having n sides	$(n-2)180°$
Distance between points on a coordinate plane	$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$
Midpoint	$\left(\frac{x_2 + x_1}{2}, \frac{y_2 + y_1}{2}\right)$
Slope of a nonvertical line (where $x_2 \neq x_1$)	$m = \frac{y_2 - y_1}{x_2 - x_1}$
Slope intercept (where m = slope, b = intercept)	$y = mx + b$
Last term of an arithmetic series Last term of a geometric series (where $n \geq 1$)	$a_n = a + (n-1)\,d$ $a_n = ar^{n-1}$
Quadratic formula	$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$
Area of a square	$A = s^2$
Volume of a cube	$V = s^3$
Area of a regular polygon	$A = \frac{1}{2}ap$ a = apothem, p = perimeter

PART III Curriculum Framework

The Arkansas Geometry Mathematics Curriculum Framework*

Strands	Content Standards	Student Learning Expectations
1. Language of Geometry (LG)	1. Students will develop the language of geometry including specialized vocabulary, reasoning, and application of theorems, properties, and postulates.	1. Define, compare, and contrast inductive reasoning and deductive reasoning for making predictions based on real-world situations. • Venn diagrams • matrix logic • conditional statements (statement, inverse, converse, and contrapositive) • figural patterns 2. Represent points, lines, and planes pictorially with proper identification, as well as basic concepts derived from these undefined terms, such as segments, rays, and angles. 3. Describe relationships derived from geometric figures or figural patterns. 4. Apply, with and without appropriate technology, definitions, theorems, properties, and postulates related to such topics as complementary, supplementary, vertical angles, linear pairs, and angles formed by perpendicular lines. 5. Explore, with and without proper technology, the relationship between angles formed by two lines cut by a transversal to justify when lines are parallel. 6. Give justification for conclusions reached by deductive reasoning. State and prove key basic theorems in geometry (i.e., the Pythagorean theorem, the sum of the measures of the angles of a triangle is 180°, and the line joining the midpoints of two sides of a triangle is parallel to the third side and half its length).
2. Triangles (T)	2. Students will identify and describe types of triangles and their special segments. They will use logic to apply the properties of congruence, similarity, and inequalities. The students will apply the Pythagorean Theorem and trigonometric ratios to solve problems in real-world situations.	1. Apply congruence (SSS ...) and similarity (AA ...) correspondences and properties of figures to find missing parts of geometric figures, and provide logical justification. 2. Investigate the measures of segments to determine the existence of triangles (triangle inequality theorem). 3. Identify and use the special segments of triangles (altitude, median, angle bisector, perpendicular bisector, and midsegment) to solve problems. 4. Apply the Pythagorean Theorem and its converse in solving practical problems. 5. Use the special right triangle relationships (30°-60°-90° and 45°-45°-90°) to solve problems. 6. Use trigonometric ratios (sine, cosine, tangent) to determine lengths of sides and measures of angles in right triangles, including angles of elevation and angles of depression. 7. Use similarity of right triangles to express the sine, cosine, and tangent of an angle, in a right triangle, as a ratio of given lengths of sides.

*The Content Standards and Student Learning Expectations listed are those that specifically relate to the items in the April 2010 End-of-Course Geometry Examination.

PART III Curriculum Framework

The Arkansas Geometry Mathematics Curriculum Framework*

Strands	Content Standards	Student Learning Expectations
3. Measurement (M)	3. Students will measure and compare, while using appropriate formulas, tools, and technology, to solve problems dealing with length, perimeter, area, and volume.	1. Calculate probabilities arising in geometric contexts. (Ex. Find the probability of hitting a particular ring on a dartboard.) 2. Apply, using appropriate units, appropriate formulas (area, perimeter, surface area, volume) to solve application problems involving polygons, prisms, pyramids, cones, cylinders, and spheres, as well as composite figures, expressing solutions in both exact and approximate forms. 3. Relate changes in the measurement of one attribute of an object to changes in other attributes. (Ex. How does changing the radius or height of a cylinder affect its surface area or volume?) 4. Use (given similar geometric objects) proportional reasoning to solve practical problems (including scale drawings). 5. Identify and apply properties of, and theorems about, parallel and perpendicular lines to prove other theorems and perform basic Euclidean constructions.
4. Relationships between Two- and Three-dimensions (R)	4. Students will analyze characteristics and properties of two- and three-dimensional geometric shapes and develop mathematical arguments about geometric relationships.	1. Explore and verify the properties of quadrilaterals. 2. Solve problems using properties of polygons. • sum of the measures of the interior angles of a polygon • interior and exterior angle measure of a regular polygon or irregular polygon • number of sides or angles of a polygon 3. Identify and explain why figures tessellate. 4. Identify the attributes of the five Platonic Solids. 5. Investigate and use the properties of angles (central and inscribed), arcs, chords, tangents, and secants to solve problems involving circles. 6. Solve problems using inscribed and circumscribed figures. 7. Use orthographic drawings (top, front, side) and isometric drawings (corner) to represent three-dimensional objects. 8. Draw, examine, and classify cross-sections of three-dimensional objects. 9. Explore non-Euclidean geometries, such as spherical geometry, and identify its unique properties which result from a change in the parallel postulate.
5. Coordinate Geometry and Transformations (CGT)	5. Students will specify locations, apply transformations, and describe relationships using coordinate geometry.	1. Use coordinate geometry to find the distance between two points, the midpoint of a segment, and the slopes of parallel, perpendicular, horizontal, and vertical lines. 2. Write the equation of a line parallel to a line through a given point not on the line. 3. Write the equation of a line perpendicular to a line through a given point. 4. Write the equation of the perpendicular bisector of a line segment. 5. Determine, given a set of points, the type of figure based on its properties (parallelogram, isosceles triangle, trapezoid). 6. Write, in standard form, the equation of a circle, given a graph on a coordinate plane or the center and radius of a circle. 7. Draw and interpret the results of transformations and successive transformations on figures in the coordinate plane. • translations • reflections • rotations (90°, 180°, clockwise and counterclockwise about the origin) • dilations (scale factor)

*The Content Standards and Student Learning Expectations listed are those that specifically relate to the items in the April 2010 End-of-Course Geometry Examination.

PART IV Item Correlation with Curriculum Framework

Released Items for Geometry*

Strands	Content Standards
1— Language of Geometry (LG)	1. Students will develop the language of geometry including specialized vocabulary, reasoning, and application of theorems, properties, and postulates.
2— Triangles (T)	2. Students will identify and describe types of triangles and their special segments. They will use logic to apply the properties of congruence, similarity, and inequalities. The students will apply the Pythagorean Theorem and trigonometric ratios to solve problems in real-world situations.
3— Measurement (M)	3. Students will measure and compare, while using appropriate formulas, tools, and technology, to solve problems dealing with length, perimeter, area, and volume.
4— Relationships between Two- and Three-dimensions (R)	4. Students will analyze characteristics and properties of two- and three-dimensional geometric shapes and develop mathematical arguments about geometric relationships.
5— Coordinate Geometry and Transformations (CGT)	5. Students will specify locations, apply transformations, and describe relationships using coordinate geometry.

Item	Strand	Content Standard	Student Learning Expectation
1	CGT	5	1
2	T	2	4
3	LG	1	5
4	LG	1	5
5	M	3	1
6	R	4	1
7	T	2	1
8	LG	1	1
9	CGT	5	5
10	M	3	2
11	T	2	5
12	R	4	7
13	M	3	5
14	LG	1	2
15	CGT	5	2
16	T	2	3
17	CGT	5	4
18	M	3	3
19	T	2	6
20	CGT	5	7
21	M	3	5
22	M	3	2
23	LG	1	6
24	R	4	8
25	T	2	2
26	R	4	2
27	LG	1	3
28	R	4	5
29	CGT	5	3
30	R	4	9
31	LG	1	4
32	M	3	4
33	R	4	6

*Only the predominant Strand, Content Standard, and Student Learning Expectation are listed for the Geometry items.

PART IV Item Correlation with Curriculum Framework

Non-Released Items for Geometry*

Strands	Content Standards
1— Language of Geometry (LG)	1. Students will develop the language of geometry including specialized vocabulary, reasoning, and application of theorems, properties, and postulates.
2— Triangles (T)	2. Students will identify and describe types of triangles and their special segments. They will use logic to apply the properties of congruence, similarity, and inequalities. The students will apply the Pythagorean Theorem and trigonometric ratios to solve problems in real-world situations.
3— Measurement (M)	3. Students will measure and compare, while using appropriate formulas, tools, and technology, to solve problems dealing with length, perimeter, area, and volume.
4— Relationships between Two- and Three-dimensions (R)	4. Students will analyze characteristics and properties of two- and three-dimensional geometric shapes and develop mathematical arguments about geometric relationships.
5— Coordinate Geometry and Transformations (CGT)	5. Students will specify locations, apply transformations, and describe relationships using coordinate geometry.

Item	Strand	Content Standard	Student Learning Expectation
1	LG	1	1
2	M	3	1
3	R	4	3
4	CGT	5	3
5	R	4	4
6	T	2	7
7	M	3	5
8	CGT	5	7
9	R	4	5
10	CGT	5	2
11	LG	1	6
12	M	3	2
13	T	2	5
14	T	2	1
15	CGT	5	1
16	M	3	4
17	T	2	3
18	CGT	5	6
19	LG	1	2
20	R	4	8
21	LG	1	3
22	M	3	3
23	T	2	4
24	T	2	2
25	LG	1	4
26	R	4	7
27	CGT	5	5
A	R	4	7
B	T	2	5
C	M	3	4
D	LG	1	5
E	CGT	5	7

*Only the predominant Strand, Content Standard, and Student Learning Expectation are listed for the Geometry items.

ACTAAP

Arkansas Comprehensive Testing, Assessment, and Accountability Program

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