ACTAAP

Arkansas Comprehensive Testing, Assessment, and Accountability Program

Released Item Booklet

Geometry
End-of-Course Examination

April 2007 Administration

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Arkansas Department of Education

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PART I Overview—April 2007

The criterion-referenced tests implemented as part of the **Arkansas Comprehensive Testing, Assessment, and Accountability Program** (ACTAAP) are being developed in response to Arkansas Legislative Act 35, which requires the State Board of Education to develop a comprehensive testing program that includes assessment of the challenging academic content standards defined by the Arkansas Curriculum Frameworks.

As part of this program, students in Arkansas public schools who had completed or were completing Geometry by the end of the spring semester participated in the *Geometry End-of-Course Examination* in April 2007.

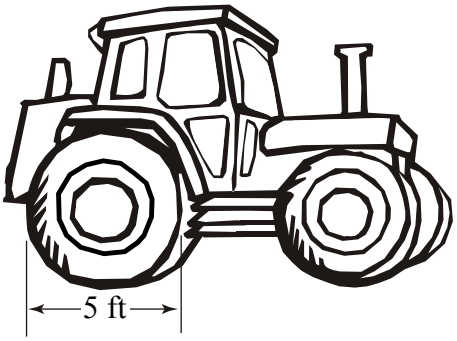
This *Released Item Booklet* for the *Geometry End-of-Course Examination* contains test questions or items that were asked of students during the April 2007 operational administration. The test items included in Part II of this booklet are those items that contributed to the student performance results for that administration.

Students were given approximately two hours each day to complete assigned test sessions during the two days of testing in April 2007. Students were permitted to use a calculator for both multiple-choice and open-response items. Students were also supplied with a reference sheet to be used so that all students would have equal access to this information during testing. (See the reference sheet on page 35 of this booklet.) All of the multiple-choice items within this booklet have the correct response marked with an asterisk (*). The open-response questions are listed with scoring guides (rubrics) immediately following. These rubrics provide information on the scoring model used for Geometry.

The development of the *Geometry End-of-Course Examination* was based on the Arkansas *Geometry Mathematics Curriculum Framework*. This framework has distinct levels: *Strands* to be taught in concert, *Content Standards* within each Strand, and *Student Learning Expectations* within each Content Standard. An abridged version of the Arkansas *Geometry Mathematics Curriculum Framework* can be found in Part III of this booklet. It is important to note that this abridged version lists only the predominant Strand, Content Standard, and Student Learning Expectation associated with each item. However, since many key concepts within the Arkansas *Geometry Mathematics Curriculum Framework* are interrelated, in many cases there are other item correlations or associations across Strands, Content Standards, and Student Learning Expectations.

Part IV of the *Released Item Booklet* contains a tabular listing of the Strand, Content Standard, and Student Learning Expectation that each question was designed to assess. The multiple-choice and open-response items found on the *Geometry End-of-Course Examination* were developed in close association with the Arkansas education community. Arkansas teachers participated as members of the Geometry Content Advisory Committee, providing routine feedback and recommendations for all items. The number of items associated with specific Strands, Content Standards, and Student Learning Expectations was based on approximate proportions suggested by the Content Advisory Committee, and their recommendations were accommodated to the greatest extent possible given the overall test design. Part IV of the *Released Item Booklet* provides Arkansas educators with specific information on how the *Geometry End-of-Course Examination* items align or correlate with the Arkansas *Geometry Mathematics Curriculum Framework* to provide models for classroom instruction.

1. The diameter of a tractor tire is 5 feet. Rounded to the nearest hundredth, how far will the tractor move when the wheel rotates once? Use $\pi = 3.14$.



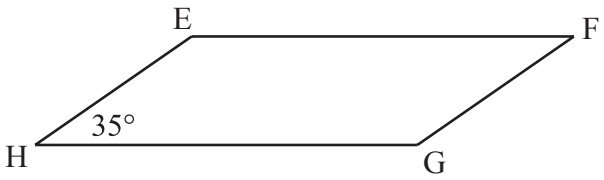
A. 7.85 feet

* B. 15.70 feet

C. 19.63 feet

D. 78.50 feet

2. What is the measure of $\angle E$ in the parallelogram below?



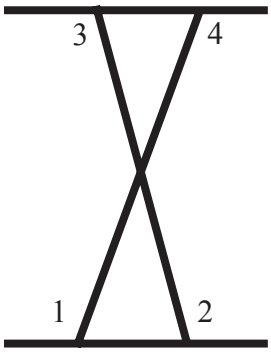
A. 35°

B. 55°

* C. 145°

D. 155°

3. An engineer designed a steel beam, shown below. The horizontal parts that form the top and bottom are parallel. To build the cross pieces, the engineer needs to know the measure of the angles shown. The measure of $\angle 1 = 110$ degrees and $m\angle 2 = 105$ degrees. What are the measures of $\angle 3$ and $\angle 4$?



A. $m\angle 3 = 70°$, $m\angle 4 = 65°$

B. $m\angle 3 = 65°$, $m\angle 4 = 70°$

C. $m\angle 3 = 110°$, $m\angle 4 = 105°$

* D. $m\angle 3 = 105°$, $m\angle 4 = 110°$

4. What is the midpoint of a segment with endpoints of (–3, 5) and (8, –17)?

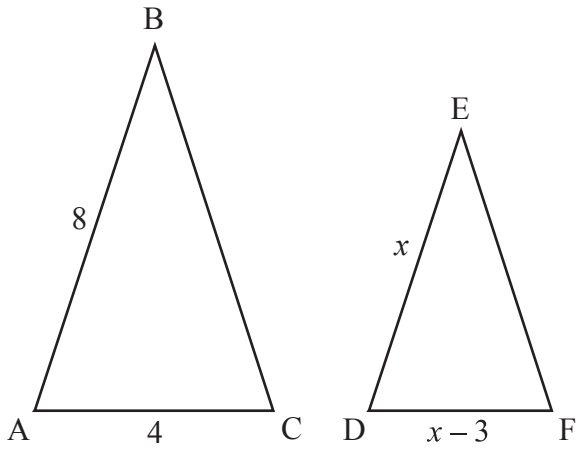
A. (3, –6)

B. (5, –12)

* C. (2.5, –6)

D. (–5.5, –9.5)

5. The triangles below are similar. What is the value of x?



A. -4

B. 3

C. 4

* D. 6

6. Lewis wrote the statements below about four constellations: Cygnus, Hercules, Leo, and Pegasus.

- The constellation Cygnus appeared after the constellation Hercules.
- The constellation Hercules appeared after the constellation Leo.
- The constellation Leo appeared before the constellation Cygnus.
- The constellation Pegasus appeared after the constellation Cygnus.

In what order did the constellations appear?

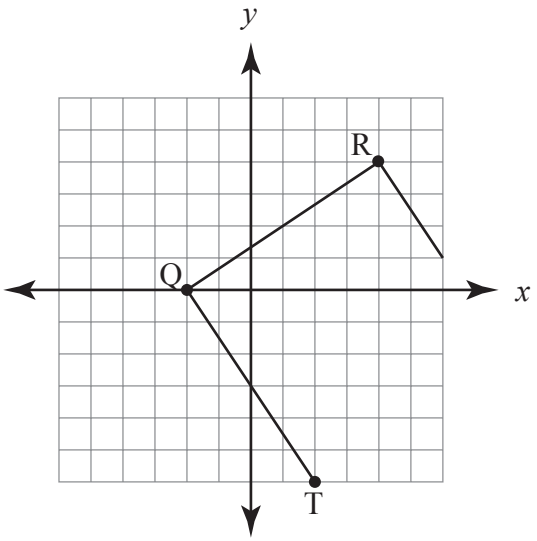
A. Cygnus, Pegasus, Leo, Hercules

B. Leo, Cygnus, Pegasus, Hercules

C. Leo, Cygnus, Hercules, Pegasus

* D. Leo, Hercules, Cygnus, Pegasus

7. A portion of rectangle QRST has been graphed on a plane.



What is the slope of line RS?

* A. $-\frac{3}{2}$

B. $-\frac{2}{3}$

C. $\frac{2}{3}$

D. $\frac{3}{2}$

8. A meter stick is held perpendicular to the ground. It forms a shadow that is 1.8 m long. At the same time, a flagpole forms a shadow that is 7.2 m long. How tall is the flagpole?

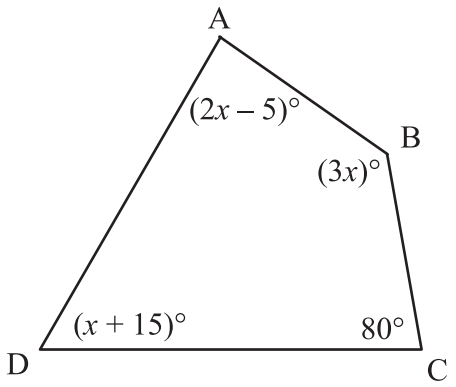
A. 0.25 m

* B. 4 m

C. 9 m

D. 12.96 m

9. Figure ABCD below is a quadrilateral. What is the value of x?



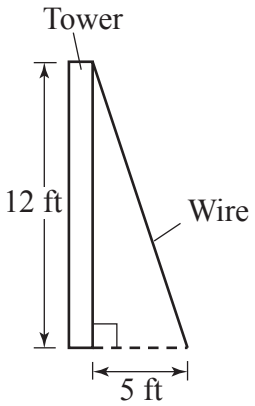
A. 15

B. 40

* C. 45

D. 65

10. A 12-foot tower is to be anchored by a wire running from the top of the tower to a point 5 feet from the bottom of the tower. What is the length of the wire?



* A. 13 ft

B. 17 ft

C. 34 ft

D. 169 ft

11. The coordinates of point A are (–2, 3). The coordinates of the midpoint of $\overline{AB}$ are (6, 1). What are the coordinates of point B?

A. (2, 2)

B. (4, –1)

C. (–4, 2)

* D. (14, –1)

12. The figure below represents Pascal's Triangle. The sum of the numbers in Row 1 is 1, the sum of Row 2 is 2, and the sum of Row 3 is 4.

Row 1				1				
Row 2			1		1			
Row 3		1		2		1		
Row 4	1		3		3		1	
Row 5	1	4		6		4		1

If this pattern continues, what is the sum of the numbers in Row 7?

A. 10

B. 32

* C. 64

D. 128

13. The circumference of a cylindrical fire hydrant is 33 inches. How many inches is the diameter of the fire hydrant, rounded to the nearest hundredth? Use $\pi = 3.14$.

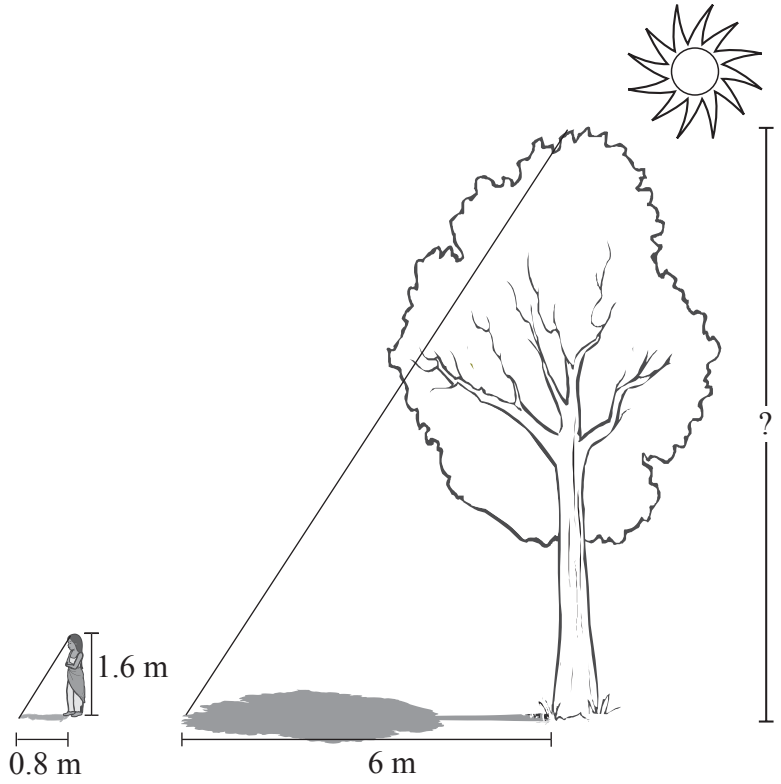
A. 3.24 inches

B. 5.25 inches

C. 6.48 inches

* D. 10.51 inches

14. Cecelia wants to find the height of a tree by measuring shadows, as shown in the figure below. The length of the tree's shadow is 6 m long, and Cecelia's shadow is 0.8 m long. Cecelia is 1.6 m tall.



What is the height of the tree, to the nearest meter?

A. 3 m

B. 8 m

C. 14 m

* D. 12 m

---

15. A line is represented by the equation $y = 12x + 33$. Which equation represents a line that is parallel to this line?

A. $y = 3x - 8$

* B. $y = 12x - 19$

C. $y = 12x + 33$

D. $y = -\frac{1}{12}x + 7$

16. Which statement is always true?

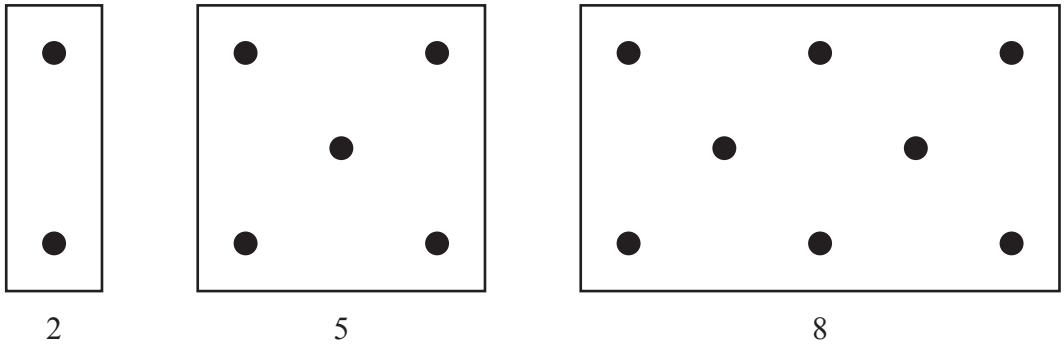
* A. All squares are rhombi.

B. All rectangles are squares.

C. All parallelograms are rectangles.

D. All quadrilaterals are parallelograms.

17. What is the 7th number in the pattern below?



A. 11
* B. 20
C. 23
D. 30

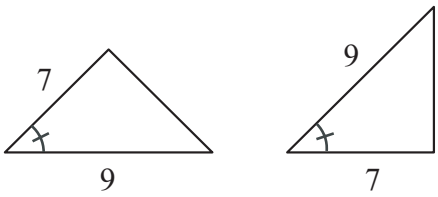
18. Lucy measures the circumference of a circular pie pan. It is 30 inches. What is the diameter of the pie pan? Use $\pi = 3.14$.

A. 3.09 inches
B. 4.78 inches
C. 6.18 inches
* D. 9.55 inches

19. Michelle is cutting pieces of wood to make shelves. Which must be true if Michelle's shelves are rectangular?

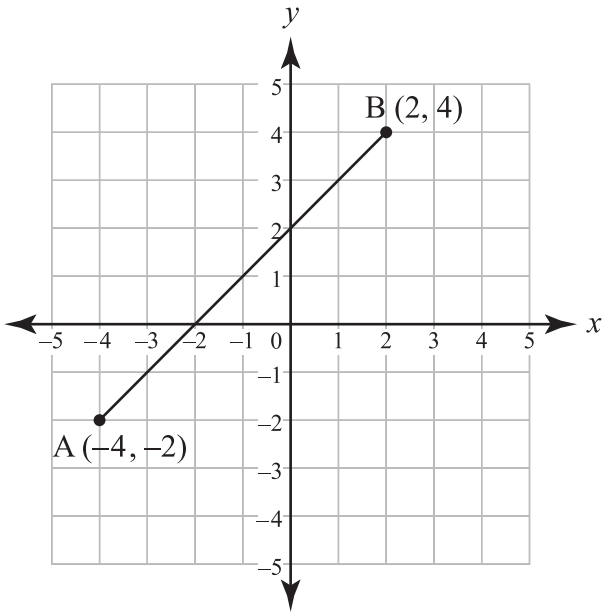
* A. The diagonals are congruent.
B. There are four congruent sides.
C. The diagonals are perpendicular.
D. The consecutive interior angles are not congruent.

20. Which theorem can be used to show that the two triangles below are congruent?



A. AAA
B. ASA
* C. SAS
D. SSS

21. Which are the coordinates of the midpoint of $\overline{AB}$ shown below?



A. (–2, 2)

* B. (–1, 1)

C. (1, –1)

D. (0, 2)

22. Ray AB is an angle bisector of ∠FAC. Which statement must be true?

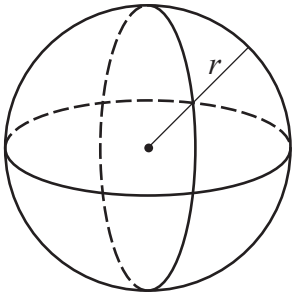
A. Angle FAB and ∠BAC each measure 45°.

B. Angle FAB and ∠BAC are complementary angles.

C. Angle FAB and ∠BAC are a linear pair.

* D. Angle FAB and ∠BAC are congruent.

23. The surface area of the sphere below is $4\pi r^2$. If the radius were to be divided by 2, how would the surface area be affected?



A. The surface area would be 16 times smaller.

B. The surface area would be 8 times smaller.

* C. The surface area would be 4 times smaller.

D. The surface area would be 2 times smaller.

24. Which **best** describes an orthographic drawing of the top view of a cone?

A. a point

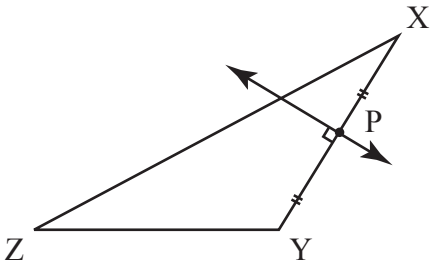
B. a sphere

C. a triangular shape

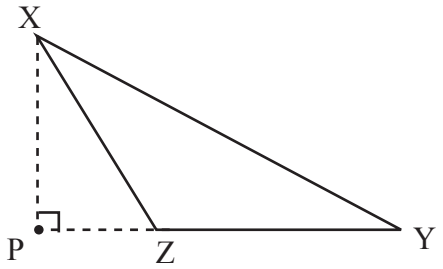
* D. a circle with a point in the center

25. In which figure is point P on a perpendicular bisector of the triangle?

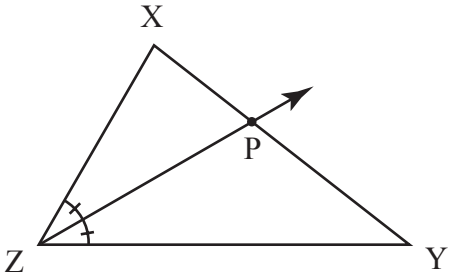
* A.



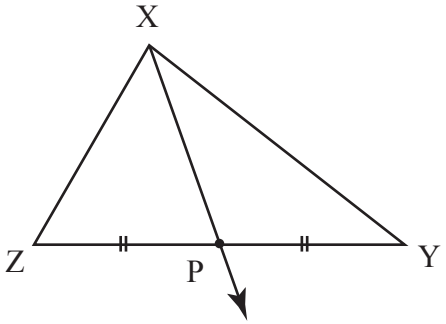
B.



C.



D.



26. What is the measure of each interior angle of a regular decagon?

A. 126°

* B. 144°

C. 162°

D. 180°

27. The equation of a circle in standard form is $(x - 3)^2 + (y + 6)^2 = 25$. What is the center point of the circle?

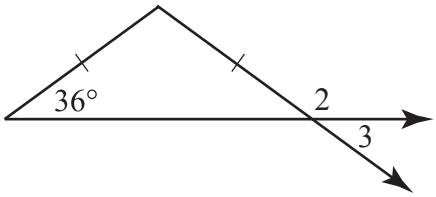
* A. (3, –6)

B. (–3, 6)

C. (6, –3)

D. (–6, 3)

28. What is the ratio of m∠2 to m∠3?



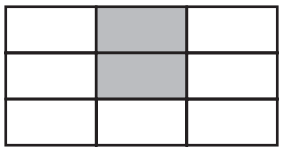
A. $\frac{36}{36}$

B. $\frac{144}{108}$

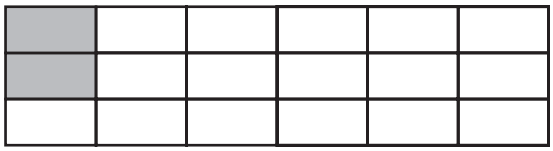
C. $\frac{108}{36}$

* D. $\frac{144}{36}$

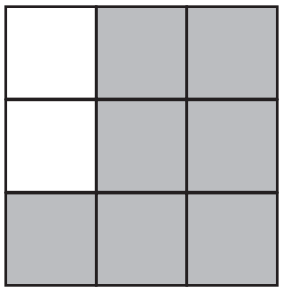
29. Below are four targets that will be used for the beanbag toss at the school carnival. Assuming the beanbag hits the target, which target has a $\frac{2}{9}$ probability of the beanbag landing on a shaded section?



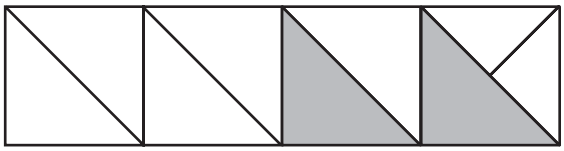
Target 1



Target 2



Target 3



Target 4

* A. Target 1

B. Target 2

C. Target 3

D. Target 4

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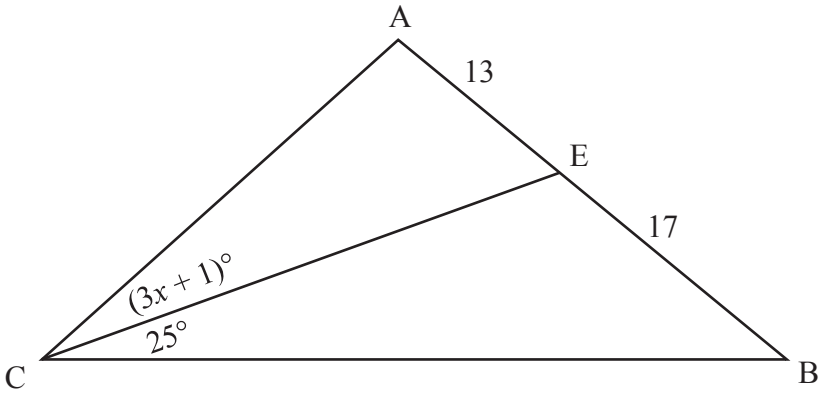
30. In the figure below, $\overline{CE}$ is an angle bisector. What is the value of x?



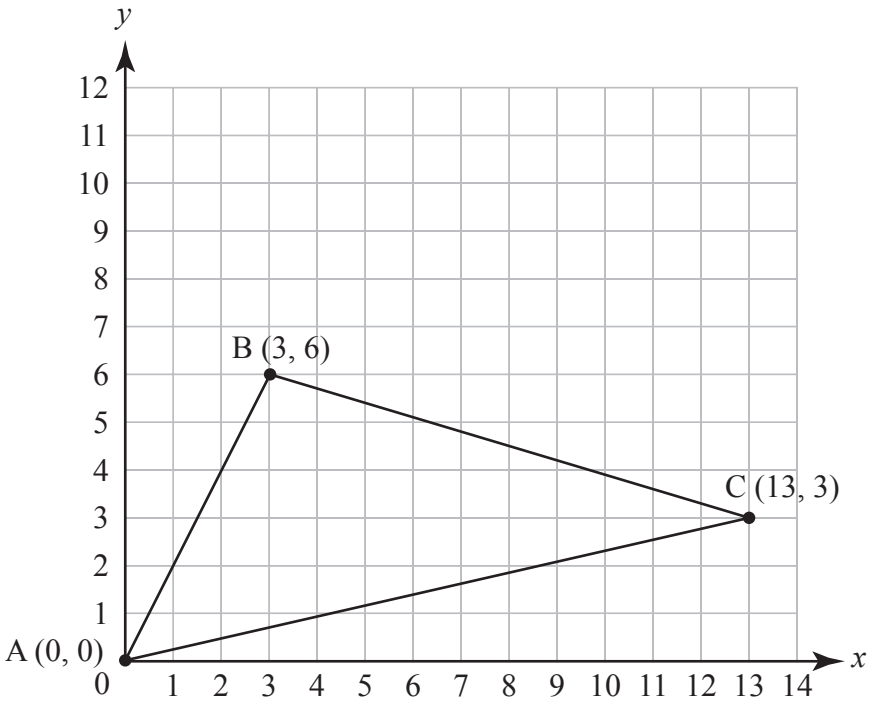
A. 4

* B. 8

C. 13

D. 25

31. What is the length of the longest side of ΔABC?



A. $\sqrt{45}$

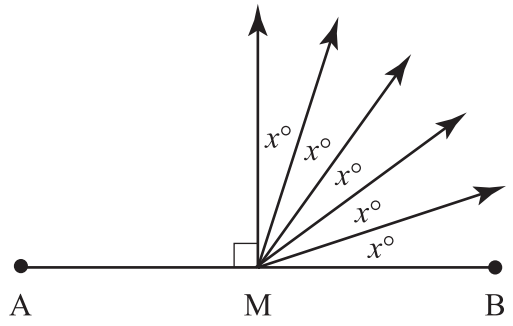
B. $\sqrt{109}$

* C. $\sqrt{178}$

D. $\sqrt{256}$

---

32. Point M is located on $\overline{AB}$. What is the value of x?



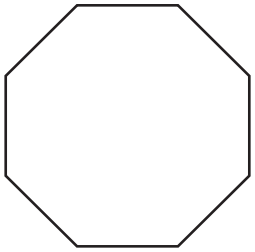
* A. 18

B. 36

C. 90

D. 180

33. The polygon below is a regular octagon. What is the measure of each exterior angle?



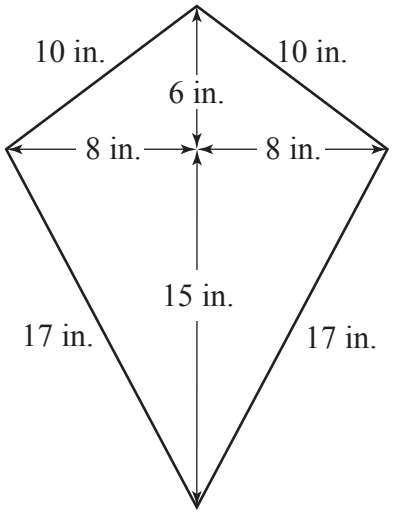
A. 40°

* B. 45°

C. 60°

D. 360°

34. Isaac's kite is shown below. What is the total area of Isaac's kite?



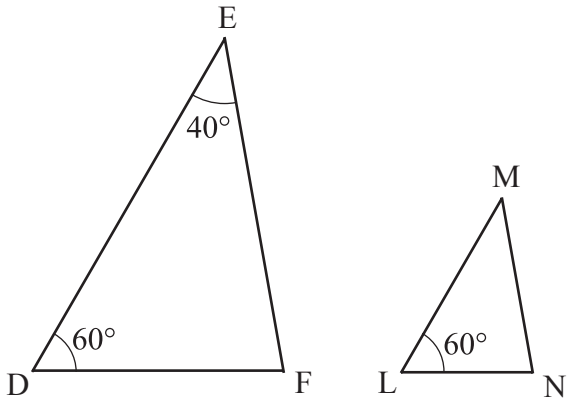
A. 91 in.^{2}

* B. 168 in.^{2}

C. 216 in.^{2}

D. 336 in.^{2}

35. What is the measure of ∠N, given that ΔDEF ~ ΔLMN as shown below?



A. 40°

B. 60°

* C. 80°

D. 100°

36. Which equation represents a line parallel to the line $y = 2x + 7$?

A. $y = -\frac{1}{2}x - 3$

B. $y = \frac{1}{2}x + 7$

C. $y = -2x + 7$

* D. $y = 2x - 3$

37. Shavon and Joe designed the target below for the water balloon toss at the school carnival. Assuming the water balloon hits the target, what is the probability that it will hit a "0"?

	1	1	1	1	1
	1	2	2	2	1
	1	2	3	2	1
0	1	2	2	2	1
	1	1	1	1	1
	0	0	0	0	0

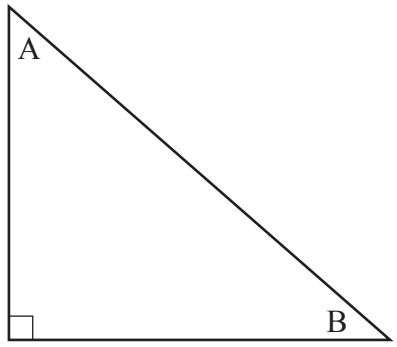
A. $\frac{2}{9}$

B. $\frac{4}{9}$

C. $\frac{6}{36}$

* D. $\frac{11}{36}$

38. In the triangle below, angles A and B may be classified as what type of angles?



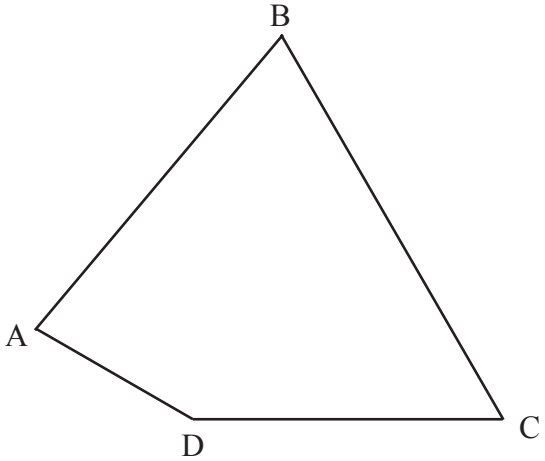
* A. complementary

B. supplementary

C. linear

D. right

39. In the figure below, m∠A = 80 degrees, m∠B = 70 degrees, and m∠D = 150 degrees. What is the measure of ∠C?



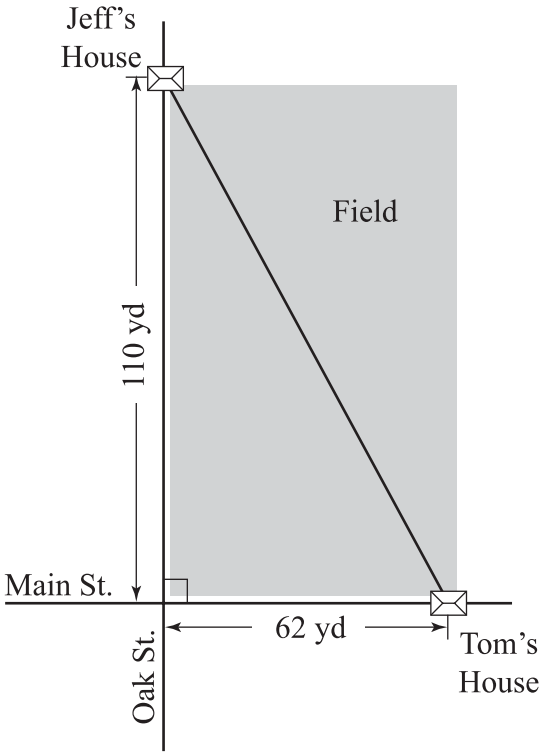
A. 50°

* B. 60°

C. 70°

D. 80°

40. Jeff lives on Oak Street, and Tom lives on Main Street.



How much farther, to the nearest yard, is it for Tom to walk down Main Street and turn on Oak Street to get to Jeff's house than if he travels the shortest distance between the houses through an empty field?

* A. 46 yd

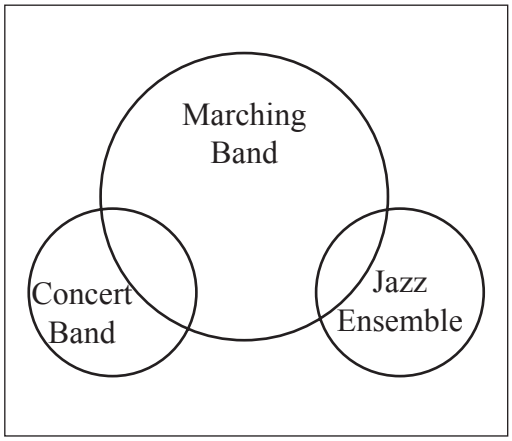
B. 48 yd

C. 126 yd

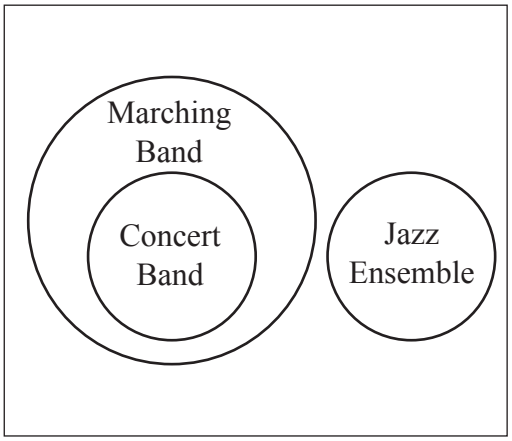
D. 172 yd

41. At a certain university, all members of the concert band are in the marching band. Some members of the jazz ensemble are also in the marching band, but none of them are in the concert band. Which Venn diagram represents the relationship described?

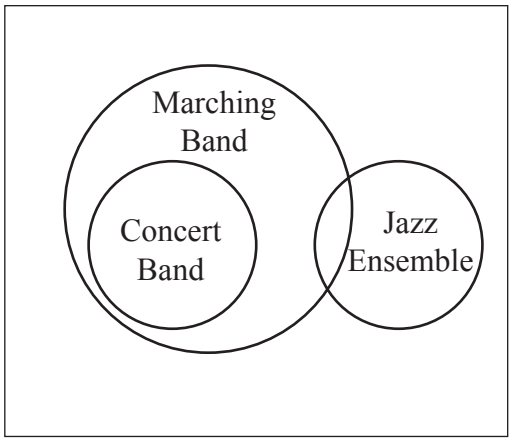
A.



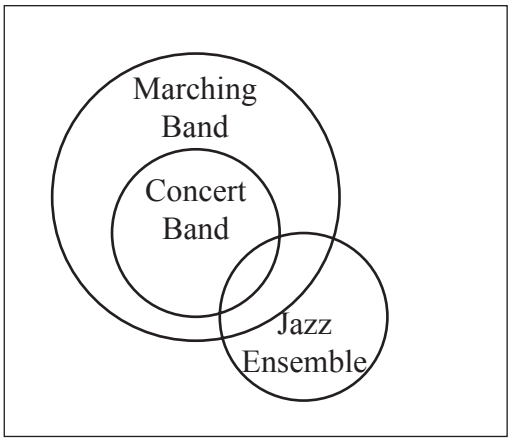
B.



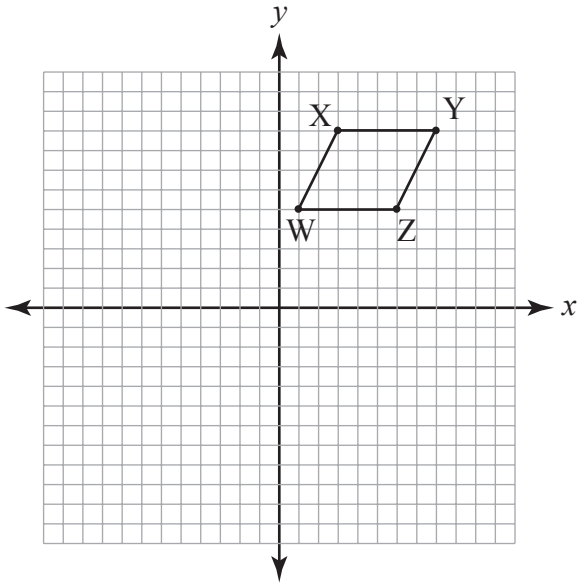
*C.



D.



42. If the parallelogram below were translated 3 units left and 6 units down, what would be the coordinates of the new image W′X′Y′Z′?



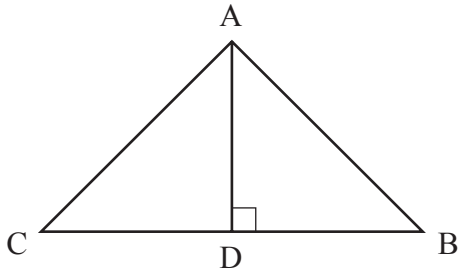
* A. W′(–2, –1), X′(0, 3), Y′(5, 3), Z′(3, –1)

B. W′(–1, –2), X′(3, 0), Y′(3, 5), Z′(–1, 3)

C. W′(4, –1), X′(6, 3), Y′(11, 3), Z′(9, –1)

D. W′(7, 8), X′(9, 12), Y′(14, 12), Z′(12, 8)

43. Segment AD is the perpendicular bisector of $\overline{BC}$. Which statement must be true?



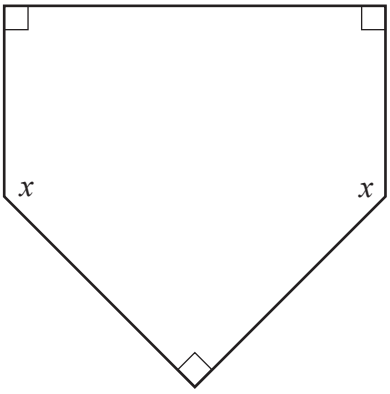
A. The triangle must be an equilateral triangle.

B. Segment AD must be half as long as $\overline{BC}$.

* C. Segment CD must be congruent to $\overline{BD}$.

D. Segment AD must be congruent to $\overline{BD}$.

44. The home plate used in a baseball game is a pentagon with three right angles. The other two angles are congruent.



What is the value of x?

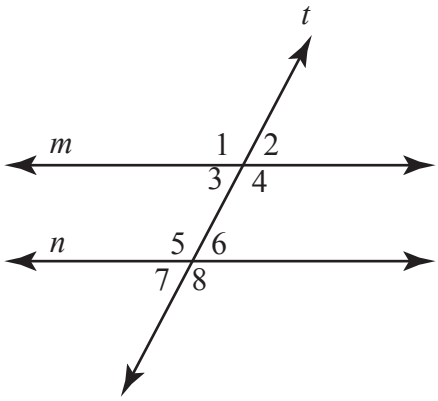
A. 108°

* B. 135°

C. 225°

D. 270°

45. Two parallel lines, m and n, are cut by transversal t, as shown in the figure below.



If $m\angle 4 = 2x - 5$ and $m\angle 6 = 3x + 20$, what is the value of x?

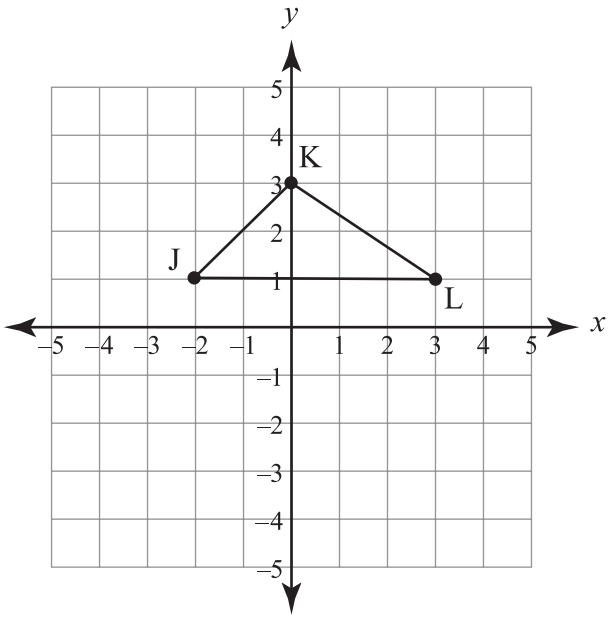
A. –25

B. 15

* C. 33

D. 165

46. Triangle JKL is translated 4 units left and 5 units up. What are the coordinates of the image of point J?



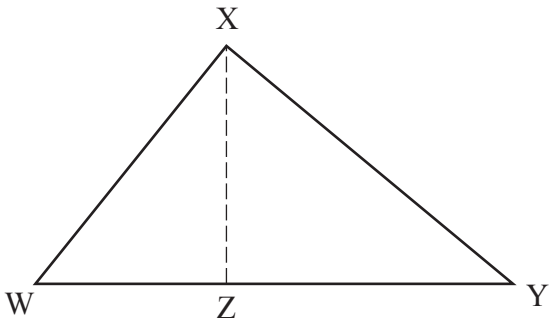
A. (2, 6)

B. (3, –3)

* C. (–6, 6)

D. (–2, 6)

47. In the figure below, m∠XZY is 90 degrees. What term accurately describes $\overline{XZ}$?



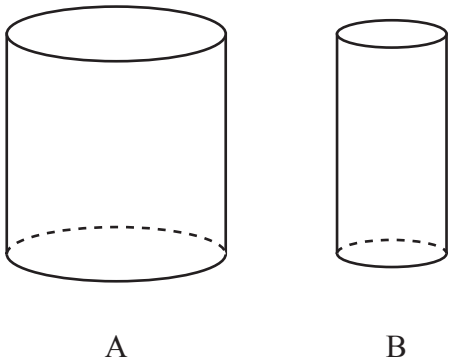
* A. altitude

B. median

C. diagonal

D. perpendicular bisector

48. Cylinders A and B have the same height. The radius of cylinder A is twice the radius of cylinder B.



How does the volume of cylinder A compare to the volume of cylinder B?

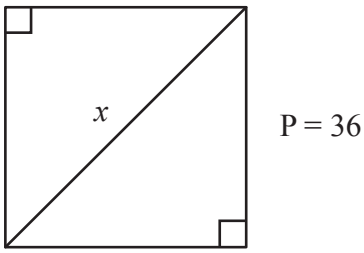
A. Cylinder A has $\frac{1}{2}$ the volume of cylinder B.

B. Cylinder A has twice the volume of cylinder B.

* C. Cylinder A has 4 times the volume of cylinder B.

D. Cylinder A has 8 times the volume of cylinder B.

49. The perimeter of the square below is 36. What is the length of the diagonal, x?



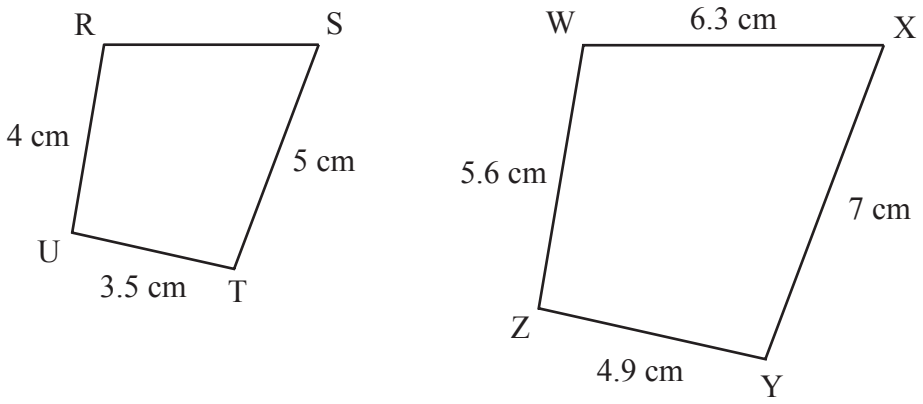
A. 6

B. 9

C. $6\sqrt{2}$

* D. $9\sqrt{2}$

50. In the figure below, RSTU ~ WXYZ.

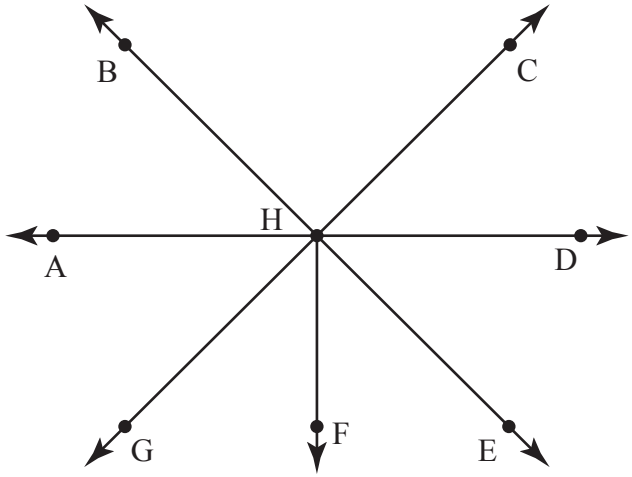


What is the length of $\overline{RS}$?

A. 4.3 cm
* B. 4.5 cm
C. 4.7 cm
D. 4.9 cm

---

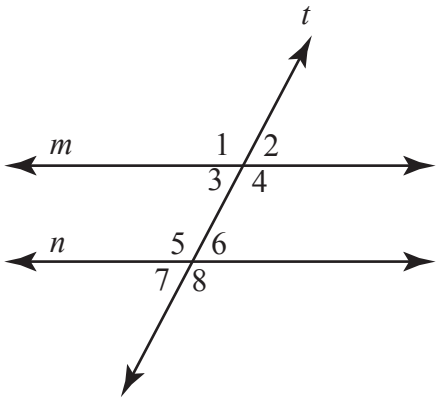
51. In the figure below, lines BE, CG, and AD, as well as ray HF, intersect at point H.



Which pair of angles must be congruent?

* A. ∠CHD and ∠AHG
B. ∠CHD and ∠EHD
C. ∠BHC and ∠GHF
D. ∠FHE and ∠EHG

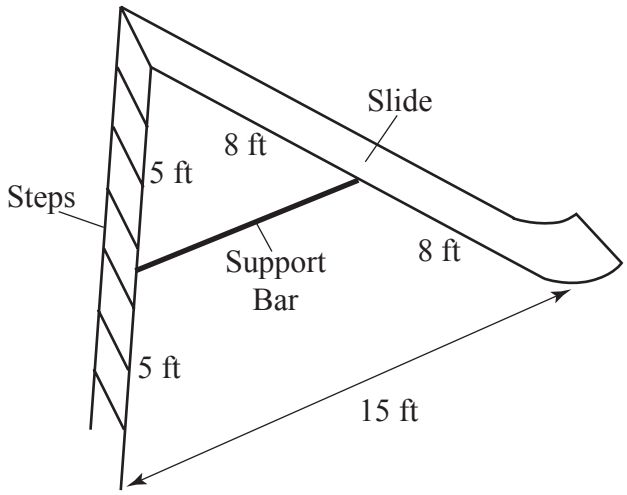
52. Two parallel lines, *m* and *n*, are cut by a transversal, *t*, as shown in the figure below.



If m∠2 = 2*x* + 7 and m∠7 = 3*x* − 13, what is the measure of ∠7?

A. 20
B. 37
* C. 47
D. 133

53. A playground slide has a support bar attached at the midpoint of the steps and the midpoint of the slide, as shown in the figure below. What is the length of the support bar?



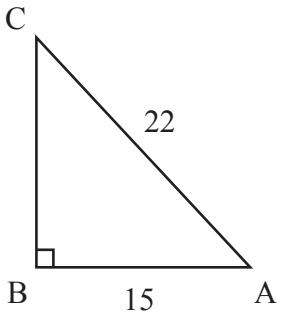
A. $6\frac{1}{2}$ ft

* B. $7\frac{1}{2}$ ft

C. 13 ft

D. 14 ft

54. What is the **approximate** measure of ∠A in the figure below?



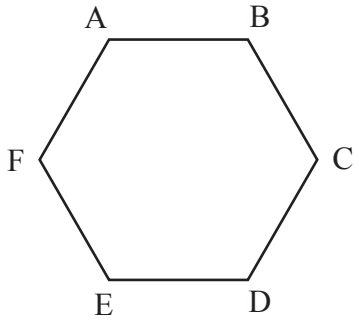
A. 30°

B. 34°

C. 43°

* D. 47°

55. ABCDEF is a regular hexagon.



What is the measure of each interior angle in ABCDEF?

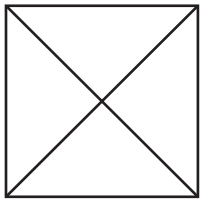
A. 60°

* B. 120°

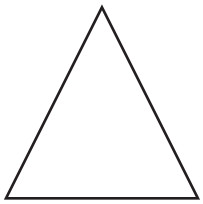
C. 180°

D. 720°

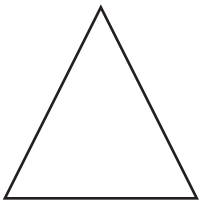
56. The top, front, and right views of a three-dimensional figure are shown below.



Top



Front



Right

What is the **correct** name for this figure?

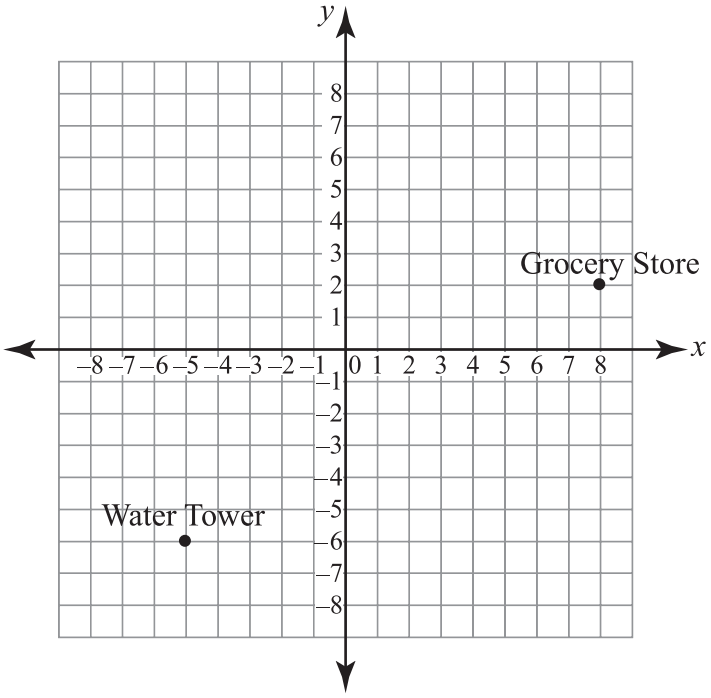
A. tetrahedron

B. square prism

* C. square pyramid

D. triangular pyramid

57. The city of Jefferson's system of street blocks is shown below.



Rounded to the nearest tenth, what is the distance from the water tower to the grocery store?

A. 10.2

* B. 15.3

C. 21.0

D. 25.0

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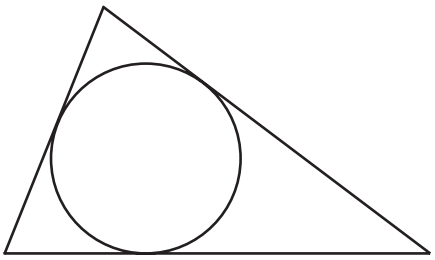
58. What is the relationship between the triangle and the circle in the figure below?



* A. The circle is inscribed in the triangle.

B. The triangle is inscribed in the circle.

C. The circle is concentric with the triangle.

D. The circle is circumscribed about the triangle.

59. Jane is designing a rectangular garden with an area of 36 ft^2. What dimensions should the garden have in order to use the **least** amount of fencing?

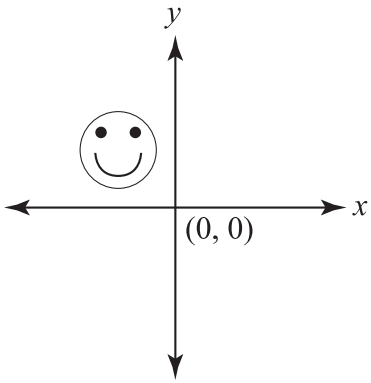
* A. 6 ft × 6 ft

B. 9 ft × 4 ft

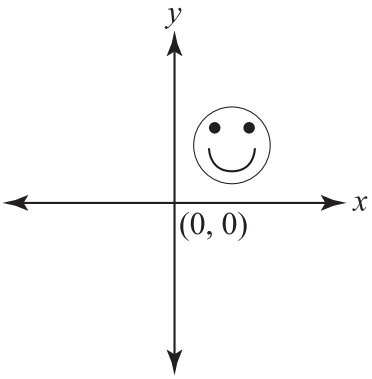
C. 12 ft × 3 ft

D. 18 ft × 2 ft

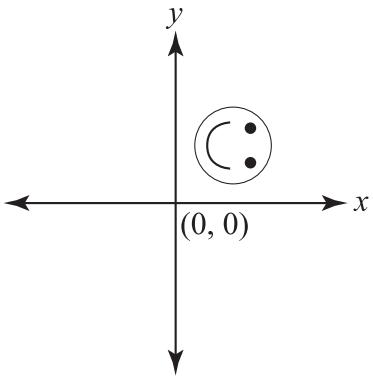
60. What would the figure below look like if it were reflected over the x-axis?



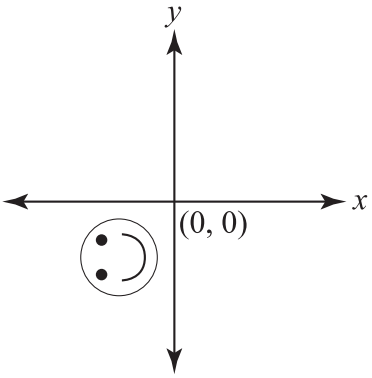
A.



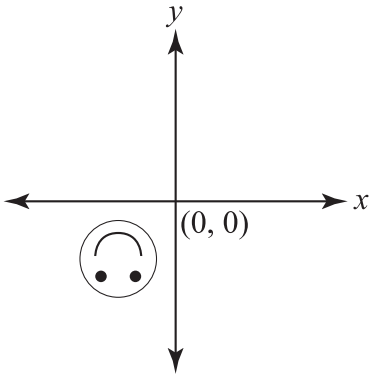
B.



C.



* D.



GEOMETRY OPEN-RESPONSE ITEM A

A. The coordinates for the vertices of quadrilateral GHIJ are given in the table below.

Vertex	Coordinates
G	(−4, 0)
H	(0, 3)
I	(4, 1)
J	(4, −4)

1. On the grid provided in your answer document, plot the vertices of quadrilateral GHIJ. Connect the vertices to form a quadrilateral.

2. Find the slopes of $\overline{GJ}$ and $\overline{HI}$ and use the slopes to make a conclusion about the relationship between the segments. Show all of your work and/or explain your answer.

3. Find the lengths of $\overline{GH}$ and $\overline{IJ}$ and use the lengths of the segments to make a conclusion about the relationship between the segments. Show all of your work and/or explain your answer.

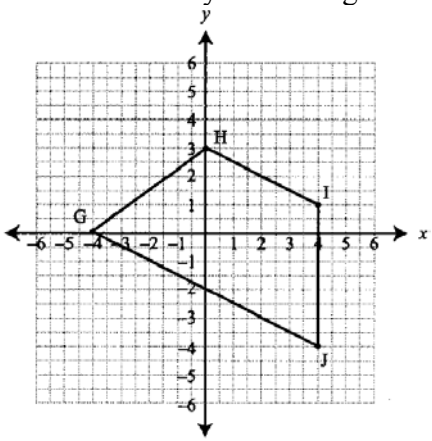
4. What type of quadrilateral is GHIJ? Be as specific as possible and use your answers from Parts 2 and 3 to support your answer.

BE SURE TO LABEL YOUR RESPONSES 1, 2, 3, AND 4.

RUBRIC FOR GEOMETRY OPEN-RESPONSE ITEM A

SCORE	DESCRIPTION
4	The student earns 5 points. The response contains no incorrect work.
3	The student earns 3 ½–4 ½ points.
2	The student earns 2–3 points.
1	The student earns ½–1 ½ points, or some minimal understanding is shown. Ex: One correct slope in Part 2 with no work shown.
0	The student earns 0 points. No understanding is shown. Ex: $IJ = 5$, if points I and J are plotted correctly with work missing.
B	Blank—No Response. A score of "B" will be reported as "NA." (No attempt to answer the item. Score of "0" assigned for the item.)

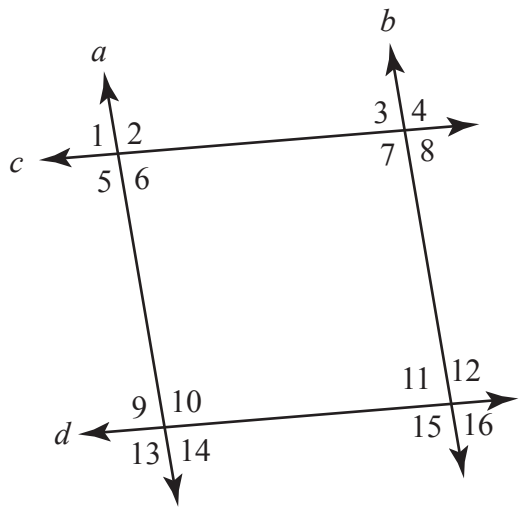
Solution and Scoring

Part	Points
1	**1 point possible** 1 point: **Correctly plotted and labeled quadrilateral *GHIJ*, as shown below.** Note: Labels may be missing but not incorrect.  OR ½ point: **Graph contains one error but is otherwise correct.** Ex: Label(s) are incorrect; points are correctly plotted. Ex: One point is off due to a counting or copy error. Ex: Intervals are inconsistent.
2	**1 point possible** 1 point: **Response includes the following:** • Both slopes are $-\frac{1}{2}$ (or equivalent). • Correct work is shown for at least one slope. Ex: Slope of $\overline{GJ} = \frac{0 - -4}{-4 - 4} = \frac{4}{-8} = -\frac{1}{2}$ Ex: Slope of $\overline{HI} = \frac{3 - 1}{0 - 4} = \frac{2}{-4} = -\frac{1}{2}$ Ex: Student uses graph to show the slope is down 1 unit and right 2 units for each segment. • Relationship: "The segments are parallel (since they have the same slope)," or "the segments have the same slope." OR ½ point: **Response is incomplete and/or contains a calculation or copy error.** Ex: Slopes are $-\frac{1}{2}$, work is shown, but the relationship statement is missing or incorrect. Ex: Slope(s) are incorrect due to a calculation or copy error. Correct procedures are used and work is shown. Relationship may be missing. Ex: Two correct slopes; work is missing. Ex: One correct slope with work; other slope is missing or incorrect. **Note: No credit is given for relationship statement if slopes are incorrect or missing.**

Part	Points
3	**2 point possible** 2 points: **Response includes the following:** • Both lengths are 5 units. • Work is correct and complete for at least *GH*: Ex: $GH = \sqrt{(-4-0)^2 + (0-3)^2} = \sqrt{4^2 + 3^2} = \sqrt{25} = 5$ $IJ = \sqrt{(4-4)^2 + (1--4)^2} = \sqrt{0^2 + 5^2} = \sqrt{25} = 5$ Ex: Student uses graph to find *GH* by creating a right triangle with *GH* as the hypotenuse and using the Pythagorean Theorem to find its length. *IJ* is found by counting the units since it is a vertical segment. • Relationship: "The sides are congruent (or they have the same length)." OR 1 point: **Response is incomplete and/or contains a calculation or copy error.** Give credit for the following or equivalent: • Lengths are 5, work is shown, but relationship statement is missing or incorrect, or • Length(s) are incorrect due to a calculation or copy error. Correct procedures are used and work is shown. Relationship may be missing, or • Both lengths are 5, work is missing; with or without relationship, or • One length is 5 with work shown, second length (and/or procedure) is missing or incorrect; with or without relationship. **Note: No credit is given for relationship statement if lengths are incorrect or missing.**
4	**1 point possible** **Note: No credit is given for "Trapezoid" without correct or equal slopes in Part 2.** **No credit is given for "Isosceles Trapezoid" without correct or equal answers (slopes & lengths) in Parts 2 and 3.** **Note: Do not give credit for "Trapezoid" if reasoning (if given) is not based on slopes or is incorrect. Do not give credit for "Isosceles Trapezoid" if reasoning (if given) is not based on slopes and lengths or is incorrect.** 1 point: **Correct (specific) name with correct and complete support.** • "Isosceles trapezoid because it has one pair of parallel sides and the other pair of sides is congruent." OR ½ point: **Give credit for any of the following:** • Correct (specific) name with missing, incomplete, or vague (but not incorrect) support. Ex: Isosceles trapezoid—one pair of sides are parallel. Ex: Isosceles trapezoid. • Correct (not specific) name. Support, if given, must be correct. Ex: Trapezoid—one pair of sides is parallel and the other pair is $\cong$. Ex: Trapezoid.

GEOMETRY OPEN-RESPONSE ITEM B

B. Lines a, b, c, and d are shown in a plane below.



1. Name one angle that must be congruent to $\angle 1$ in order to prove that $a \parallel b$. Explain your reasoning.

2. Name one angle that must be congruent to $\angle 1$ in order to prove that $c \parallel d$. Explain your reasoning.

3. If $a \parallel b$ and $c \parallel d$, what is the relationship between $\angle 9$ and $\angle 4$? Explain your reasoning.

4. The measure of $\angle 13$ equals $(6x + 40)$ degrees and the measure of $\angle 16$ equals $(120 - 4x)$ degrees. If $a \parallel b$, what is the value of x? Show your work and/or explain your answer.

BE SURE TO LABEL YOUR RESPONSES 1, 2, 3, AND 4.

RUBRIC FOR GEOMETRY OPEN-RESPONSE ITEM B

SCORE	DESCRIPTION
4	The student earns 5 points. The response contains no incorrect work.
3	The student earns 3 ½–4 ½ points.
2	The student earns 2–3 points.
1	The student earns ½–1 ½ points, or some minimal understanding is shown. Ex: Correct angle is given in Parts 1 and 2 with incorrect explanation. Ex: Correct relationship is given in Part 3 with incorrect explanation.
0	The student earns 0 points. No understanding is shown.
B	Blank—No Response. A score of "B" will be reported as "NA." (No attempt to answer the item. Score of "0" assigned for the item.)

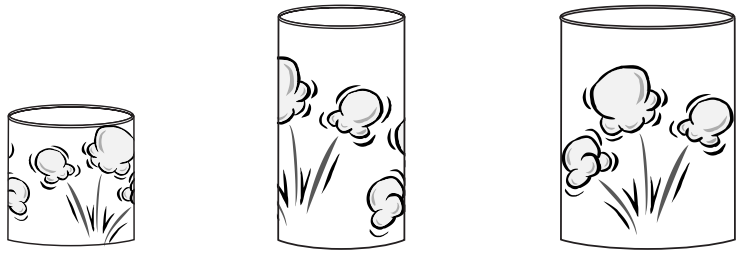
Solution and Scoring

Part	Points
1	**1 point possible** 1 point: **Correct answer ($\angle 3$ or $\angle 8$) with correct explanation.** Give credit for the following or equivalent: • "$\angle 3$; If two lines are cut by a transversal and the corresponding angles are congruent, then the lines are parallel," or • "$\angle 8$; If two lines are cut by a transversal and the alternate exterior angles are congruent, then the lines are parallel." Note: The converse of either of the above theorems is acceptable, or • "Angle 3. They are corresponding angles," or • "$\angle 8$; Because it's the opposite exterior angle," or • " Angle 8 because it's vertical to angle 3 and 1 and 3 are $\cong$." OR ½ point: **One correct answer with vague or missing, but not incorrect, explanation.** Give credit for the following or equivalent: Ex: "Angle 3. It's in the same place…" Ex: "Angle 8 because it's a vertical angle." Ex:"$\angle 3$."
2	**1 point possible** 1 point: **One correct answer ($\angle 9$ or $\angle 14$) with correct explanation.** Give credit for the following or equivalent: • "$\angle$ 9; If two lines are cut by a transversal and the corresponding angles are congruent, then the lines are parallel," or • "$\angle 14$; If two lines are cut by a transversal and the alternate exterior angles are congruent, then the lines are parallel." Note: The converse of either of the above theorems is acceptable, or • "Angle 9. They are corresponding angles," or • "$\angle 14$; Because it's the opposite exterior angle," or • "Angle 14 because it's vertical to angle 9 and 1 and 9 are $\cong$." OR ½ point: **Correct answer with vague or missing, but not incorrect, explanation.** Give credit for the following or equivalent: Ex: "Angle 9 since if you shifted it over they're the same…" Ex: "Angle 14 because it's vertical." Ex: "$\angle$ 9."

Part	Points
3	**1 point possible** 1 point: **Correct relationship with correct and complete explanation.** Note: You must be able to follow the logic step by step. Names of angle relationships are not required. Give credit for the following or equivalent: Ex: Angle 9 and Angle 4 are supplementary: Angle 9 $\cong$ Angle 1 $\cong$ Angle 3 Angle 3 and Angle 4 are supplementary Therefore, Angle 9 and Angle 4 are supplementary. Ex: "Angles 9 & 4 are supplementary because 3 and 4 are supplementary" (Angle 3 $\cong$ Angle 1 $\cong$ Angle 9 given in Parts 1 & 2) Ex: "9 and 4 are supplementary: $\angle 10 \cong \angle 2 \cong \angle 4$, 9 & 10 are supplementary and 3 & 4 are supplementary, so 9 & 4 are too." OR ½ point: **Correct answer with incomplete, vague, or missing, but not incorrect, explanation.** Give credit for the following or equivalent: Ex: "9 and 4 are supplementary since 10 $\cong$ 4." Ex: "9 and 4 are supplementary because they add up to 180 degrees." Ex: "9 & 4 add up to 180 º."
4	**2 points possible** 1 point: **Correct value for x: 10.** Note: Do not give credit if procedure is incorrect AND 1 point: **Correct and complete procedure shown and/or explained.** Work may contain a calculation or copy error Give credit for the following or equivalent: • $(6x + 40) + (120 - 4x) = 180$ (initial equation is adequate) $2x + 160 = 180$ $2x = 20$ $x = \#$.

GEOMETRY OPEN-RESPONSE ITEM C

C. A movie theater sells popcorn in 3 different sizes of cylindrical tubs.



	Small	Medium	Large
Cost	$2.50	$4.50	$8.25
Radius	3 inches		
Height	6 inches		
Volume			
Number of Servings	2 servings		

1. In your answer document, copy and complete the table above using the following information. Show all of your work and/or explain your answer.

   - The medium tub has the same radius as the small tub, but it holds 1.5 times as much popcorn as the small tub.
   - The radius of the large tub of popcorn is 1.5 times the radius of the small tub. The height of the large tub is 1.5 times the height of the small tub.

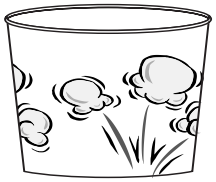
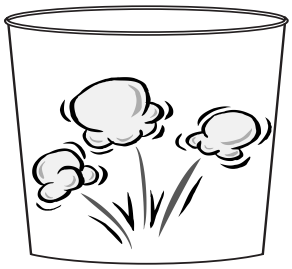
2. A family of 6 people goes to the movies. What is the **least** expensive way to purchase a serving of popcorn for each person? Show all of your work and/or explain your answer.

BE SURE TO LABEL YOUR RESPONSES 1 AND 2.

RUBRIC FOR GEOMETRY OPEN-RESPONSE ITEM C

SCORE	DESCRIPTION
4	The student earns 5 points. The response contains no incorrect work. Units are not required, but must be correct if included in student work.
3	The student earns 3 ½–4 ½ points.
2	The student earns 2–3 points.
1	The student earns ½–1 ½ points, or some minimal understanding is shown.
0	The student earns 0 points. No understanding is shown.
B	Blank—No Response. A score of "B" will be reported as "NA." (No attempt to answer the item. Score of "0" assigned for the item.)

Solution and Scoring

	 Small	Medium	 Large
Cost	**$2.50**	**$4.50**	**$8.25**
Radius	3 in. (Given)	**3 (in.)**	**4.5 (in.)** (1.5 x 3 may be missing)
Height	6 in. (Given)	**9 (in.)** $\frac{255}{(3^2)\pi}$ (work may be missing depending upon method)	**9 (in.)** (1.5 x 6 may be missing)
Volume	**54π or 169 – 170(in.^{3})** **$V = \pi(3^2)6$**	**81π or 253.5 – 255 (in.^{3})** **V = 170(1.5) or** **$V = \pi(3^2)9$**	**182.25π or 572 – 573 (in.^{3})** **$V = \pi(4.5^2)(9)$**
Number of Servings	2 servings (Given)	**3 servings** $\frac{170}{2} = \mathbf{85}$, $\frac{255}{85} = \mathbf{3}$ or equivalent (may be missing)	**6, 6.75, or 7 servings (may be rounded or truncated)** $\frac{170}{2} = \mathbf{85}$, $\frac{573}{85} = \mathbf{6.75}$ or equivalent

Part	Points
1	**4 points possible** **<u>Radii, Heights, Volumes</u>** 3 points: **Correct radii, heights, and volumes for all three sizes.** Correct procedure shown for finding at least one volume. Work for finding radii and heights is not required. OR 2 points: **Give credit for the following:** • Correct radii, heights, and volumes for all three sizes with no work shown, or • Answer(s) are incorrect due to one calculation error with correct procedure used and work shown. OR 1 ½ points: **Give credit for the following:** • Correct radii, heights, and volumes for two of the three sizes with correct procedure used and work shown for at least one volume. OR 1 point: **Give credit for the following:** • Correct radii, heights, and volumes for two of the three sizes with no work shown, or • Correct procedures shown (volume) for two out of three sizes using correct radii and heights. **<u># of Servings</u>** <u>Note: Number of servings must be based on volumes that are given</u>. 1 point: **Correct number of servings (based on volumes) for Medium and Large sizes.** • Correct procedure used and work is shown for Large size, or • Proportional reasoning is shown for at least Medium size. OR ½ point: **Give credit for the following:** • Correct # of servings (based on volumes) for Medium and Large with no work shown, or • Number of serving(s) incorrect due to a calculation error with correct procedure used and work shown. Note: To check number of servings, divide volume by 85 or V(small) ÷ 2. Answer may be rounded.

Part	Points
2	**1 point possible** 1 point: **Give credit for the following or equivalent:** • Least expensive choice (based on number of servings given for the three sizes) with correct and complete procedure (three options for comparison). Ex: Three Small: $7.50, 2 Medium: $9.00, 1 Large: $8.25, so they should buy 3 small tubs. Ex: Cost per serving: Small: $2.50/2 servings = $1.25 per serving Medium: $4.50/3 servings = $1.50 per serving Large: $8.25/6 = $1.375 per serving Three small tubs would be the least expensive. Ex: Cost per serving: Small: $2.50/2 servings = $1.25 per serving Medium: $4.50/3 servings = $1.50 per serving Large: $8.25/6.75 servings = $1.22 per serving A large container is the least expensive. OR ½ point: **Response is incomplete or contains one calculation error.** Give credit for the following or equivalent: • Least expensive choice (based on number of servings given for the three sizes) with work shown for two options for comparison, or • Correct and complete procedure (three options)—least expensive choice is missing or incorrect. Note: Do not give credit for least expensive choice with only one option listed.

GEOMETRY OPEN-RESPONSE ITEM D

D. Jason, Patrick, and Bryce are learning to weld metal in their Industrial Technology class. Their project is to weld together metal rods of **different** lengths in order to form triangles. Their teacher gives each boy 4 rods of varying lengths, as shown in the table below.

Name	**Lengths of Rods (in inches)**			
Jason	5	3	12	4
Patrick	16	15	4	9
Bryce	7	2	10	19

Each boy can choose 3 of his rods to weld together, but they must form a triangle.

1. Explain the theorem that the boys can use in order to determine which 3 of their 4 rods should be welded together to make the triangle.

2. For each boy, determine one possible combination of his 3 metal rods that can be welded together to form the triangle. Show that this combination obeys the theorem from Part 1. If there are no combinations for any one of the boys, provide work to support your conclusion.

BE SURE TO LABEL YOUR RESPONSES 1 AND 2.

RUBRIC FOR GEOMETRY OPEN-RESPONSE ITEM D

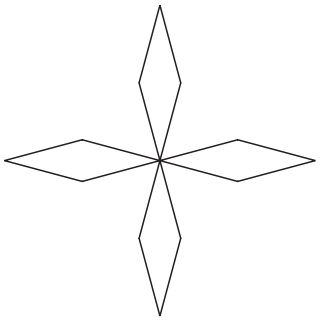
SCORE	DESCRIPTION
4	The student earns 4 points. The response contains no incorrect work. Units are not required, but must be correct if included in student work.
3	The student earns 3–3 ½ points.
2	The student earns 2–2 ½ points.
1	The student earns ½–1 ½ points, or some minimal understanding is shown.
0	The student earns 0 points. No understanding is shown.
B	Blank—No Response. A score of "B" will be reported as "NA." (No attempt to answer the item. Score of "0" assigned for the item.)

Solution and Scoring

Part	Points
1	**1 point possible** 1 point: **Correct name, statement of, or explanation of theorem.** Give credit for the following or equivalent: • The Triangle Inequality Theorem, or • "The sum of the lengths of **any** two sides of a triangle is greater than the length of the third side" (give credit if "any" is omitted), or • "Take the two shortest lengths and add them together. That number must be bigger than the third length," or • "The length of a side of a triangle must be between the sum and difference of the other two lengths." Note: No credit for the Pythagorean Theorem or its statement.
2	**3 points possible** Note: No credit will be awarded in Part 2 for answer or procedure if support is completely invalid (obviously incorrect procedure) in Parts 1 and/or 2 with no correct procedure shown. Ex: "Jason could use 3-, 4-, and 5-inch rods because they're closest together." (True, but insufficient.) Ex: "Patrick could use 4-, 15-, and 16-inch rods because you could make a right triangle." (Not true.) ½ point: **Correct combination for Jason: 3-, 4-, and 5-inch rods.** AND ½ point: **Correct and complete procedure shown.** Work may contain a calculation or copy error if answer is incorrect. Give credit for the following or equivalent: • $5 + 3 > 4$, $5 + 4 > 3$, $3 + 4 > 5$, or • $3 + 4 > 5$, or • $5 + 4 = 9 > 3$ and $5 – 4 = 1 < 3$. (If using this method, it is not necessary to check all three pairs.) Note: If student uses the Pythagorean Theorem in Part 1, then credit will be awarded for 3, 4, 5 with or without correct procedure shown. AND ½ point: **1 correct combination for Patrick: 4-, 15-, and 16-inch rods or 9-, 15-, and 16-inch rods.** AND ½ point: **Correct and complete procedure shown for appropriate combination.** Work may contain a calculation or copy error if answer is incorrect. Give credit for the following or equivalent: • $4 + 15 > 16$, $4 + 16 > 15$, $15 + 16 > 4$, or • $9 +15 > 16$, $9 + 16 > 15$, $15 + 16 > 9$, or • $4 + 15 > 16$ or $9 + 15 > 16$, or • $4 + 16 = 20 > 15$ and $16 – 4 = 12 < 15$, or • $16 + 15 = 31 > 9$ and $16 – 15 = 1 < 9$. AND ½ point: **Correct response for Bryce: No combination works.** AND ½ point: **Give credit for the following or equivalent:** • All 4 combinations are checked: 7, 2, 10 : $7 + 2 < 10$ 7, 2, 19 : $7 + 2 < 19$ 2, 10, 19 : $2 + 10 < 19$ 7, 10, 19 : $7 + 10 < 19$, or • The smallest three numbers and the largest three numbers are checked as follows: 7, 2, 10: $7 + 2 < 10$ (Work for 7, 2 ,19 is not required.) 7, 10, 19: $7 + 10 < 19$ (Work for 2, 10, 19 is not required.)

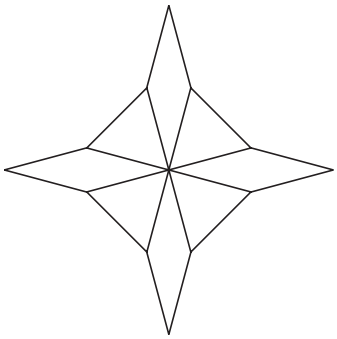
GEOMETRY OPEN-RESPONSE ITEM E

E. Jessica is making a window ornament from pieces of stained glass in the shapes of rhombi and triangles. Jessica begins the ornament with four rhombus-shaped pieces arranged at equal rotations around a center point, as shown below.



1. The smallest angles of each rhombus piece measure 30 degrees. In your answer document, draw one of the rhombi pieces and label all of its angle measures. Show all of your work and/or explain your answer.

Next, Jessica places triangular pieces between the rhombi pieces, connecting the vertices formed by the largest angles of each rhombus, as shown below.



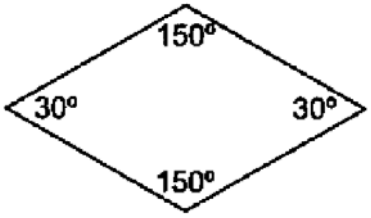
2. In your answer document, draw one of the triangular pieces. Label all of its angle measures. Show all of your work and/or explain your answer.

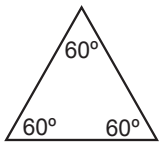
BE SURE TO LABEL YOUR RESPONSES 1 AND 2.

RUBRIC FOR GEOMETRY OPEN-RESPONSE ITEM E

SCORE	DESCRIPTION
4	The student earns 5 points. The response contains no incorrect work. The label of "degrees" or "°" is included somewhere in the response. An explanation of why the measures of the center angles of one triangle + one rhombus = 90° is included in Part 2.
3	The student earns 4 points.
2	The student earns 2–3 points.
1	The student earns 1 point, or some minimal understanding is shown.
0	The student earns 0 points. No understanding is shown.
B	Blank—No Response. A score of "B" will be reported as "NA." (No attempt to answer the item. Score of "0" assigned for the item.)

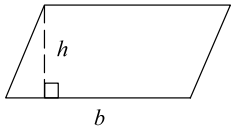
Solution and Scoring

Part	Points
1	**2 points possible** 1 point: **Drawing of a rhombus with correct and complete angle measures as shown below.** Note: The "degree" label is required only for a score of 4.  AND 1 point: **Correct and complete procedure shown and/or explained.** Work may contain a calculation or copy error. Give credit for the following or equivalent: • (All angles in a quadrilateral must add to 360º) $30 + 30 + x + x = 360$, $60 + 2x = 360$ $2x = 300$, $x = \#$, or • $(n - 2)180 = (4 - 2)180 = 360$ $360 - 60 = 300$, $300/2 = \#$, or • "In a rhombus, consecutive angles are supplementary, so I know the other angle is 150 degrees."

Part	Points
2	**3 points possible** 1 point: Drawing of an equilateral triangle with correct and complete angle measures as shown below. Note: The "degree" label is required only for a score of 4.  AND 1 point: **Correct and complete procedure shown and/or explained finding the measure of the angle of the triangle whose vertex is the center point.** Work may contain a calculation or copy error. Give credit for the following or equivalent: • (All angles about the center point must add to 360º) x = measure of angle of triangle whose vertex is the center point 30 + 30 + 30 + 30 + x + x + x + x = 360 120 + 4x = 360 4x = 240 x = 60, or • 2(30) + 2x = 180 2x = 120 x = 60, or • "The angles of the rhombus and the triangle whose vertex is the center point make a 90º angle." (Explanation of why this is true is required for a score of 4.) "So the measure of the angle of the triangle is 90 – 30 = 60º." AND 1 point: **Statement or drawing showing that the <u>triangle is isosceles</u> and the base angles measure 60º (work may be missing).** Work may contain a calculation or copy error or may be based on an incorrect measure of the "center angle" (work must be shown to receive credit). Give credit for the following or equivalent: • "<u>Because the sides shared with the rhombi are congruent, the triangles are isosceles</u>. The base angles must be congruent and the sum of the angles in the triangle must be 180°. I subtract 60 from 180 and get 120. Half of 120 is 60 for each base angle," or • Diagram shows that 2 sides are of the triangle ≅ and the base angles measure 60º (with "center angle" measuring 60º), or • "The <u>triangle is isosceles</u>. Since the 1st angle is 60º, I know the base angles are each 60º since 60 + 60 + 60 = 180, the triangle is equilateral." **Note: Do not give credit if the response jumps to the conclusion that the triangle is equilateral (or marks 3 sides ≅ on a diagram) without initially stating or showing that 2 sides are ≅.** Ex: "Measure of base angles = 60º since 180/3 = 60" Ex: "It's an equilateral triangle so all angles are 60 degrees." (Does not initially state that the triangle is isosceles.)

End-of-Course Mathematics Reference Sheet

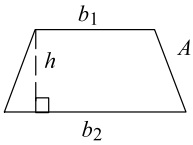
Parallelogram



$P =$ sum of all sides

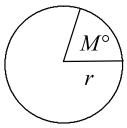
$A = bh$

Trapezoid



$A = \frac{h(b_1 + b_2)}{2}$

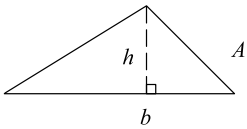
Arc and Sector



Arc Length $= \left(\frac{M}{360}\right) \times 2\pi r$

Sector area $= \left(\frac{M}{360}\right) \times \pi r^2$

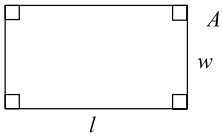
Triangle



$P =$ sum of all sides

$A = \frac{bh}{2}$

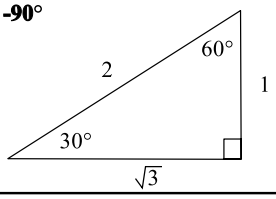
Rectangle



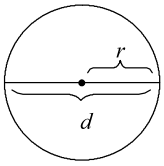
$P = 2l + 2w$

$A = lw$

30° -60° -90°



Circle



$C = 2\pi r$

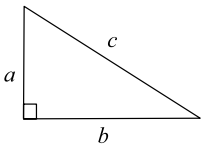
$C = \pi d$

$A = \pi r^2$

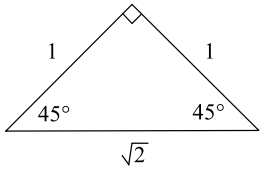
$\pi \approx 3.14$

Pythagorean Theorem

$a^2 + b^2 = c^2$

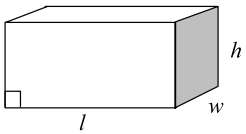


45° -45° -90°



Rectangular Solid

Volume $= lwh$

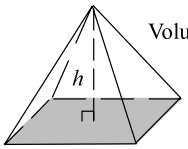


Surface area $= 2lw + 2lh + 2wh$

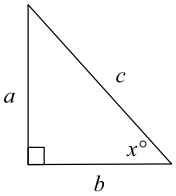
Pyramid

$B =$ *area of base (shaded)*

Volume $= \frac{Bh}{3}$



Trigonometric Ratios

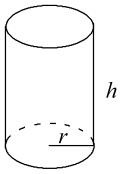


$\sin x° = \frac{a}{c}$

$\cos x° = \frac{b}{c}$

$\tan x° = \frac{a}{b}$

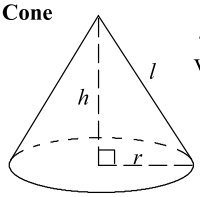
Cylinder



Volume $= \pi r^2 h$

Surface area $= 2\pi rh + 2\pi r^2$

Cone

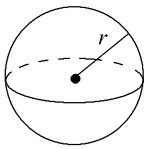


$l =$ slant height

Volume $= \frac{\pi r^2 h}{3}$

Surface area $= \pi rl + \pi r^2$

Sphere



Volume $= \frac{4\pi r^3}{3}$

Surface area $= 4\pi r^2$

Miscellaneous Formulas

Area of an equilateral triangle	$A = \frac{s^2\sqrt{3}}{4}$ $s =$ length of a side
Distance	rate × time
Interest	principal × rate × time in years
Sum of the angles of a polygon having n sides	$(n - 2)180°$
Distance between points on a coordinate plane	$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$
Midpoint	$\left(\frac{x_2 + x_1}{2}, \frac{y_2 + y_1}{2}\right)$
Slope of a nonvertical line (where $x_2 \neq x_1$)	$m = \frac{y_2 - y_1}{x_2 - x_1}$
Slope intercept (where m = slope, b = intercept)	$y = mx + b$
Last term of an arithmetic series Last term of a geometric series (where $n \geq 1$)	$a_n = a + (n - 1)d$ $a_n = ar^{n-1}$
Quadratic formula	$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$
Area of a square	$A = s^2$
Volume of a cube	$V = s^3$
Area of a regular polygon	$A = \frac{1}{2}ap$ $a =$ apothem, $p =$ perimeter

The Arkansas *Geometry Mathematics Curriculum Framework**

Strands	Content Standards	Student Learning Expectations
1—LANGUAGE OF GEOMETRY (LG)	1. Students will develop the language of geometry including specialized vocabulary, reasoning, and application of theorems, properties, and postulates.	1. Define, compare, and contrast inductive reasoning and deductive reasoning for making predictions based on real-world situations: • Venn diagrams • matrix logic • conditional statements (statement, inverse, converse, and contrapositive) 3. Describe relationships derived from geometric figures or figural patterns. 4. Apply, with and without appropriate technology, definitions, theorems, properties, and postulates related to such topics as complementary, supplementary, vertical angles, linear pairs, and angles formed by perpendicular lines. 5. Explore, with and without appropriate technology, the relationship between angles formed by two lines cut by a transversal to justify when lines are parallel.
2—TRIANGLES (T)	2. Students will identify and describe types of triangles and their special segments. Students will use logic to apply the properties of congruence, similarity, and inequalities. Students will apply the Pythagorean Theorem and trigonometric ratios to solve problems in real-world situations.	1. Apply congruence (SSS ...) and similarity (AA ...) correspondences and properties of figures to find missing parts of geometric figures and provide logical justification. 2. Investigate the measures of segments to determine the existence of triangles (triangle inequality theorem). 3. Identify and use the special segments of triangles (altitude, median, angle bisector, perpendicular bisector, and midsegment) to solve problems. 4. Apply the Pythagorean Theorem and its converse in solving practical problems. 5. Use the special right triangle relationships (30°-60°-90° and 45°-45°-90°) to solve problems. 6. Use trigonometric ratios (sine, cosine, tangent) to determine lengths of sides and measures of angles in right triangles, including angles of elevation and angles of depression.
3—MEASUREMENT (M)	3. Students will measure and compare, while using appropriate formulas, tools, and technology, to solve problems dealing with length, perimeter, area, and volume.	1. Calculate probabilities arising in geometric contexts. (Ex. Find the probability of hitting a particular ring on a dartboard.) 2. Apply, using appropriate units, appropriate formulas (area, perimeter, surface area, volume) to solve application problems involving polygons, prisms, pyramids, cones, cylinders, and spheres, as well as composite figures, expressing solutions in both exact and approximate forms. 3. Relate changes in the measurement of one attribute of an object to changes in other attributes. (Ex. How does changing the radius or height of a cylinder affect its surface area or volume?) 4. Use (given similar geometric objects) proportional reasoning to solve practical problems (including scale drawings).

*The Content Standards and Student Learning Expectations listed are those that specifically relate to the released test items in this booklet.

The Arkansas *Geometry Mathematics Curriculum Framework** (continued)

Strands	Content Standards	Student Learning Expectations
4—RELATIONSHIPS BETWEEN TWO AND THREE DIMENSIONS (R)	4. Students will analyze characteristics and properties of two- and three-dimensional geometric shapes and develop mathematical arguments about geometric relationships.	1. Explore and verify the properties of quadrilaterals. 2. Solve problems using properties of polygons: • sum of the measures of the interior angles of a polygon • interior and exterior angle measure of a regular polygon or irregular polygon • number of sides or angles of a polygon 6. Solve problems using inscribed and circumscribed figures. 7. Use orthographic drawings (top, front, side) and isometric drawings (corner) to represent three-dimensional objects.
5—COORDINATE GEOMETRY AND TRANSFORMATIONS (CGT)	5. Students will specify locations, apply transformations, and describe relationships using coordinate geometry.	1. Use coordinate geometry to find the distance between two points, the midpoint of a segment, and the slopes of parallel, perpendicular, horizontal, and vertical lines. 2. Write equations of lines in slope-intercept form and use slope to determine parallel and perpendicular lines. 3. Determine, given a set of points, the type of figure based on its properties (parallelogram, isosceles triangle, trapezoid). 4. Write, in standard form, the equation of a circle, given a graph on a coordinate plane or the center and radius of a circle. 5. Draw and interpret the results of transformations and successive transformations on figures in the coordinate plane: • translations • reflections • rotations (90°, 180°, clockwise and counterclockwise about the origin) • dilations (scale factor)

*The Content Standards and Student Learning Expectations listed are those that specifically relate to the released test items in this booklet.

PART IV Geometry Item Correlation with Curriculum Framework—April 2007

Released Items for Geometry*

Strands	Content Standards
1— LANGUAGE OF GEOMETRY (LG)	1. Students will develop the language of geometry including specialized vocabulary, reasoning, and application of theorems, properties, and postulates.
2— TRIANGLES (T)	2. Students will identify and describe types of triangles and their special segments. Students will use logic to apply the properties of congruence, similarity, and inequalities. Students will apply the Pythagorean Theorem and trigonometric ratios to solve problems in real-world situations.
3— MEASUREMENT (M)	3. Students will measure and compare, while using appropriate formulas, tools, and technology, to solve problems dealing with length, perimeter, area, and volume.
4— RELATIONSHIPS BETWEEN TWO- AND THREE-DIMENSIONS (R)	4. Students will analyze characteristics and properties of two- and three-dimensional geometric shapes and develop mathematical arguments about geometric relationships.
5— COORDINATE GEOMETRY AND TRANSFORMATIONS (CGT)	5. Students will specify locations, apply transformations, and describe relationships using coordinate geometry.

Item	Strand	Content Standard	Student Learning Expectation
1	M	3	2
2	R	4	1
3	LG	1	5
4	CGT	5	1
5	T	2	1
6	LG	1	1
7	CGT	5	1
8	M	3	4
9	R	4	1
10	T	2	4
11	CGT	5	1
12	LG	1	3
13	M	3	2
14	T	2	1
15	CGT	5	2
16	R	4	1
17	LG	1	3
18	M	3	2
19	R	4	1
20	T	2	1
21	CGT	5	1
22	LG	1	4
23	M	3	2
24	R	4	7
25	T	2	3
26	R	4	2
27	CGT	5	4
28	LG	1	4
29	M	3	1
30	T	2	3
31	CGT	5	1
32	LG	1	4
33	R	4	2

Item	Strand	Content Standard	Student Learning Expectation
34	M	3	2
35	T	2	1
36	CGT	5	2
37	M	3	1
38	LG	1	4
39	R	4	1
40	T	2	4
41	LG	1	1
42	CGT	5	5
43	T	2	3
44	R	4	2
45	LG	1	5
46	CGT	5	5
47	T	2	3
48	M	3	3
49	T	2	5
50	M	3	4
51	LG	1	4
52	LG	1	5
53	M	3	4
54	T	2	6
55	R	4	2
56	R	4	7
57	CGT	5	1
58	R	4	6
59	M	3	3
60	CGT	5	5
A	CGT	5	3
B	LG	1	5
C	M	3	3
D	T	2	2
E	R	4	1

*Only the predominant Strand, Content Standard, and Student Learning Expectation is listed for the Geometry items.

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