

# Targeted Mathematics Topics

## Targeted Mathematics Professional Development Opportunities by Appropriate Grade Levels

K	1	2	3	4	5	6	7	8	9	10	11	12
Linear Measurement, available Summer 2013												
Developing Fact Fluency, available Summer 2012												
Problem Situations: Addition and Subtraction and Nature of "Equals", available Summer, 2012												
Problem Situations: Multiplication and Division and Nature of "Equals", available Summer 2012												
Strategies, Algorithms, and Recording Systems: Multi-digit Addition and Subtraction, available Summer 2012												
Non-geometric Measurement, available Summer 2013												
Developing the Base Ten, available Summer 2012												
Fraction Concepts: Equal Sharing, available Summer 2012												
Numerical Data, Available summer 2013												
Area, Surface Area, and Volume Measurement, available Summer 2013												
Angles, Coordinate Grids and Scale, available Summer 2013												
Strategies, Algorithms, and Recording Systems: Multi-digit Multiplication and Division, available												
Other Problem Situations: Multiplication and Division, available Summer 2012												
Fraction Operations: Multiple Grouping, available Summer 2012												
Algebraic Thinking as a Bridge to Functions, available Summer 2013												
Developing Proportional Reasoning, available Summer 2012												
Fraction Operations: Partial Groups, available Summer 2013												
Geometry, available Summer 2013												
Data Modeling Part One: Inventing Displays, Center and Precision, available Summer 2012												
Data Modeling Part Two: Chance, Modeling and Inference, available Summer 2013												
Geometric Measurement, available Summer 2013												
Functions, available Summer 2013												

## **Mathematics Content Professional Development Course Guide ADE CCSSM Professional Development Project (Updated 2/6/2013)**

Each course is listed in the following way:

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**Course Title** (Titles were created to clearly indicate mathematics content)

**Recommended Grades**

*(Face-to-Face: Season and Year first available to be offered in traditional face-to-face workshops; Online: Season and Year first available to be offered as an online Moodle course)*

Course Description

Prerequisites or relationships to other courses

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Flow charts showing recommended or possible paths through the professional development courses are provided.

Course offerings, in both face-to-face workshop form and online Moodle form, will be made by cooperatives and university STEM centers through their usual workshop registration systems. Contact the Teacher Center Coordinator or mathematics specialist at your local cooperative to learn more.

Access to training materials for districts to use in-house, for professional development or professional learning community interactions, can be gained by contacting the mathematics specialist at your local cooperative or university STEM Center. Materials are password protected to protect videos containing students from use by the general public.

## Mathematics Content Professional Development Courses

### CATEGORY: Number, Operations, and Algebraic Thinking

#### **Problem Situations: Addition and Subtraction and the Nature of “Equals” Grades K-3**

(Face-to-Face: *Summer 2012*; Online: *Winter 2012*)

This course will investigate the different types of addition and subtraction situations and children’s understanding of the equal sign. The topics include:

- The structure of the different problem situations
- How the problems are alike and different
- How children’s thinking about the problem situations is different than the way adults think about it
- Children’s strategies and level of development
- Connections to operations and algebraic thinking
- Instructional implications

This course is a prerequisite for the courses *Developing Fact Fluency and Strategies, Algorithms and Recording Systems: Multi-digit Addition and Subtraction*. Year 1 or more of CGI can also serve as the prerequisite.

#### **Problem Situations: Multiplication and Division and the Nature of “Equals” Grades K-6**

(Face-to-Face: *Summer 2012*; Online: *Winter 2012*)

This course will investigate the following problems situations: multiplication (unknown product) and division (group size unknown and number of groups unknown) that are foundational for the fractions, the base-ten numbers system, facts fluency and multi-digit multiplication and division. We will also analyze children’s understanding of the equal sign and the role it plays in developing children’s understanding of multiplication and division. These include:

- The structure of the different problem situations
- How the problems are alike and different
- How children’s thinking about the problem situations is different than the way adults think about it
- Children’s strategies and level of development
- Connections to operations and algebraic thinking
- Instructional implications

Finally, the course will demonstrate how these concepts form the foundation for so many other mathematical concepts, and thus why it is a prerequisite for several other courses.

This course is a prerequisite for the courses *Developing Fact Fluency; Developing the Base Ten System; Strategies, Algorithms and Recording Systems: Multi-digit Multiplication and Division*; and all fractions courses. Year 1 or more of CGI can also serve as the prerequisite.

### **Developing Fact Fluency Grades K-3**

(Face-to-Face: *Summer 2012*; Online: *Winter 2012*)

This class will focus on the development of children's understanding of single digit operations using manipulatives/pictures to applying properties of operations to solve fact problems in addition, subtraction, multiplication and division. We will study children's development, how to press students to make symbolic connections to their informal strategies and focus on (and generalize) the properties of operations that are in play in their work. We will ultimately define what it means for a student to be fluent and the steps to facilitate students' growth to that end.

The course *Problem Situations: (Either Addition and Subtraction OR Multiplication and Division) and Nature of "Equals"* is required before you are eligible to participate in this course. Year 1 or more of CGI can also serve as the prerequisite.

### **Developing the Base Ten System Grades K-6**

(Face-to-Face: *Summer 2012*; Online: *Winter 2012*)

In this class, we will make sense of the base 10 system generally through number and properties of operations. We will begin by looking at the roots that begin in kindergarten based in the counting sequence, to generalizing place value understanding for multi-digit whole numbers in fourth grade, and finally making connections for 5<sup>th</sup> and 6<sup>th</sup> grade to the decimal system. This class is based on the multiplicative structure of the base ten system and properties of operations with the only difference between grade levels being the magnitude of the numbers.

The course *Problem Situations: Multiplication and Division and the Nature of "Equals"* is required before you are eligible to participate in this course.

This course is a prerequisite for the courses *Strategies, Algorithms and Recording Systems: Multi-digit Addition and Subtraction* and *Strategies, Algorithms and Recording Systems: Multi-digit Multiplication and Division*. Year 1 or more of CGI can also serve as the prerequisite.

## **Other Problem Situations: Multiplication and Division Grades 3-6**

(Face-to-Face: *Summer 2012*; Online: *Winter 2012*)

This course will investigate the 3 different types of multiplication and division situations (equal grouping, multiplicative comparison and area/array). These include:

- The structure of the different problem situations
- How the problems are alike and different
- How children’s thinking about the problem situations is different than the way adults think about it
- A beginning examination of understanding the implications of using area/array problems in developing multiplicative understanding.
- Children’s strategies and level of development
- Connections to operations and algebraic thinking
- Instructional implications

The course *Problem Situations: Multiplication and Division and the Nature of “Equals”* is required before you are eligible to participate in this course. Year 1 or more of CGI can also serve as the prerequisite.

## **Fraction Concepts: Equal Sharing Grades 1-6**

(Face-to-Face: *Summer 2012*; Online: *Winter 2012*)

This course uses as a required text the book Extending Children’s Mathematics: Fractions and Decimals by Susan B. Empson and Linda Levi. The sessions will delve into the mathematics available for instruction when students engage in equal sharing problems. Attention is given to:

- Understanding and characterizing student approaches
- Identifying the mathematics embedded in student work
- Using student work to further learning about fractions, such as-
  - Equivalence
  - Notation
  - What constitutes a whole
  - Addition of fractions
- Recording mathematical thinking and attending to properties of operations.

The course *Problem Situations: Multiplication and Division and the Nature of “Equals”* is required before you are eligible to participate in this course. Year 1 or more of Thinking Mathematically or Extending Children’s Mathematics can serve as the prerequisite.

This course is a prerequisite for the course *Fraction Operations: Multiple Grouping*.

### **Fraction Operations: Multiple Grouping Grades 3-6**

(Face-to-Face: *Summer 2012*; Online: *Winter 2012*)

This course uses as a required text the book Extending Children's Mathematics: Fractions and Decimals by Susan B. Empson and Linda Levi. The sessions will explore the mathematics and strategies raised when students tackle "multiple grouping" problems, problems "involving a whole number of equal groups of fractional amounts." The course includes:

- Examining how students think about and solve multiple grouping problems
- Identifying the mathematics embedded in student work
- Using student work to increase understanding of fractions and operations, such as-
  - Composing, decomposing, and recomposing wholes
  - Understanding  $1/b$  and its relationship to  $a/b$  (repeated addition or multiplication)
  - Relating multiplication and division
- Recording mathematical thinking and attending to properties of operations.

The course *Fraction Concepts: Equal Sharing* is required before you are eligible to participate in this course. Year 1 or more of CGI can also serve as the prerequisite.

### **Fraction Operations: Partial Groups Grades 4-7**

(Face-to-Face: *Summer 2013*; Online: *Fall 2013*)

This course uses as a required text the book Extending Children's Mathematics: Fractions and Decimals by Susan B. Empson and Linda Levi. The sessions will explore the mathematics and strategies for multiplying and dividing fractions in a variety of problem situations. The course includes:

- Examining how students think about and solve partial groups problems;
- Identifying the mathematics embedded in student work;
- Using student work to increase understanding of fractions and operations; and

- Thinking about how the inherent “division” included in a fraction affects the operations.

The course will also bridge fraction and decimal computation.

This course requires two pre-requisite courses from this series: *Fraction Concepts: Equal Sharing* and *Fraction Operations: Multiple Grouping*. Both must be completed before you are eligible to participate in this course.

### **Strategies, Algorithms and Recording Systems: Multi-digit Addition and Subtraction Grades K-4**

(Face-to-Face: *Summer 2012*; Online: *Winter 2012*)

In this class, we will focus on children’s development of strategies from manipulatives/pictures to applying properties of operations to solve multi-digit addition and subtraction problems. This class will build off of the *Problem Situations: Addition and Subtraction and the Nature of “Equals”* for K-3 and look at how strategies build from single digit facts to multi-digit numbers using similar strategies and properties of operations. There will be an emphasis on mathematizing and symbolizing students’ informal strategies to help them to become flexible and fluent in their application.

The courses *Problem Situations: Addition and Subtraction and the Nature of “Equals”* and *Developing the Base Ten System* are required before you are eligible to participate in this course.

### **Strategies, Algorithms and Recording Systems: Multi-Digit Multiplication and Division Grades 3-6**

(Face-to-Face: *Summer 2012*; Online: *Winter 2012*)

In this class, we will focus on children’s development of strategies from manipulatives/pictures to applying properties of operations to solve multi-digit addition and subtraction problems. This class will build off of the *Problem Situations: Multiplication and Division and the Nature of “Equals”* for K-6 and look at how strategies build from single digit facts to multi-digit numbers using similar strategies and properties of operations. There will be an emphasis on mathematizing and symbolizing students’ informal strategies to help them to become flexible and fluent in their application.

The courses *Problem Situations: Multiplication and Division and the Nature of “Equals”* and *Developing the Base Ten System* are required before you are eligible to participate in this course.

## **CATEGORY: Geometry and Measurement**

### **Linear Measurement**

#### **Grades K-3**

(Face-to-Face: *Summer 2013*; Online: *Fall 2013*)

This course will investigate the big ideas behind linear measurement, how children think about these ideas, the developmental trajectory of their thinking, and what this means instructionally. This course will include:

- Investigating the big ideas of linear measurement with emphasis on zero point, point of origin, the unit, iteration, and partitions of units;
- Examining and making sense of students' thinking as it relates to the big ideas;
- Analyzing students' learning trajectory using the Theory of Measurement construct map;
- Reviewing tasks and experiences designed to help students develop deep conceptions of linear measurement that transcend other measurement concepts and mathematical concepts;
- Making connections behind the big ideas of linear measurement to standard units and measuring tools; and
- Exploring connections of partitioning units to computation and the properties of operations.

This course sets a foundation for all numerical data courses and all other measurement courses in this series.



## **Non-geometric Measurement Grades K-5**

(Face-to-Face: *Summer 2013*; Online: *Fall 2013*)

This course will investigate the development of non-geometric measurements in grades K-5, beginning in grade K with describing measureable attribute and extending thru grade 5 with conversions between units. Topics include:

- Time
- Weight and mass
- Capacity
- Money
- Other non-geometric measures
- Related rates

The course *Linear Measurement – Grades K-3* is recommended for completion before taking this course because the non-geometric measures can all be superimposed on a linear model.

## **Area, Surface Area and Volume Measurement Grades 3-5**

(Face-to-Face: *Summer 2013*; Online: *Fall 2013*)

This course will focus on the big ideas of area, surface area and volume. We will examine the relationships between the three and how children’s thinking evolves related to these big ideas. This course includes:

- Making sense of the big ideas of measure (iteration, tiling, the privileged unit – square and cubic units, and partial units) –
  - The visualization of iterating of a row or column through an area,
  - The visualization of iterating an area through a volume, and
  - Mathematizing students’ visualization and represent their thinking using mathematical notation and properties of operations;
- Examining and making sense of students thinking as it relates to the big ideas;
- Analyzing students’ learning trajectory; and
- Reviewing tasks and experiences designed to help students develop deep conceptions of measurement.

This course does not have a pre-requisite, but will align with other courses in this series for K-5 teachers on geometric measures: *Linear Measurement – Grades K-3* and *Angles, Coordinate Grids and Scale – Grades 3-5*.

## **Angles, Coordinate Grids and Scale Grades 3-5**

(Face-to-Face: *Summer 2013*; Online: *Fall 2013*)

This course will focus on making sense of the different interpretations of angle with emphasis on angle as the degree of turn. We will also make connections to geometry by exploring the properties of shape and form as they relate to angles. These will be integrated through mapping directions using scale and the coordinate grid. This course includes:

- Exploring angle through movement (pacing and turning) using non-standard units;
- Translating non-standard units of angle into standards units;
- Analyzing shape through walking polygons (exploring angle and the implication on the shapes of quadrilaterals); and
- Mapping movements using written directions of units and angles and using coordinate system.

This course does not have a pre-requisite, but will align with other courses in this series for K-5 teachers on geometric measures: *Linear Measurement – Grades K-3* and *Area, Surface Area and Volume Measurement – Grades 2-5*.

## **Geometric Measurement Grades 6-12**

(Face-to-Face: *Summer 2013*; Online: *Fall 2013*)

This course will investigate the conceptual development of geometric measurements found in grades 6-12 Common Core State Standards for Mathematics including:

- Distances on rectangular coordinate systems including Pythagorean Relationships and Trigonometric Relationships
- Perimeters, Circumferences and Arc Lengths
- Areas of 2-dimensional figures
- Relationships between perimeter (circumference) and area of 2-dimensional figures.
- Surface area of polyhedrons, cones, cylinders and spheres
- Volume of polyhedrons, cones, cylinders and spheres
- Relationship between linear, area and volume measures in similar figures.
- Angle relationships including supplementary, complementary, vertical, adjacent, angles formed by parallel lines cut by a transversal, sums of interior and exterior angles of polygons, etc.

This course does not have a pre-requisite, but will align with other courses in this series for K-5 teachers on geometric measures: *Linear Measurement –*

*Grades K-3; Area, Surface Area and Volume Measurement – Grades 2-5; and Angles, Coordinate Grids and Scale – Grades 3-5.*

## **Geometry Grades 5-12**

(Face-to-Face: *Summer 2013*; Online: *Fall 2013*)

This course uses a required text the book Developing Essential Understanding of Geometry: Grades 6-8 from NCTM. Although the text is written around grades 6-8 the ideas will be extended through high school. The course will investigate the development of geometric thinking around big ideas and essential understandings. The big ideas include:

- Behind every measurement formula lies a geometric result.
- Geometric thinking involves developing, attending to, and learning how to work with imagery.
- A geometric object is a mental object that, when constructed, carries with it traces of the tools by which it was constructed.
- Classifying, naming, defining, posing, conjecturing, and justifying are codependent activities in geometric investigation.

This course does not have a pre-requisite but will align with two other courses in this series: *Geometric Measurement – Grades 6-12* and *Geometry – Grades K-5*.

## **CATEGORY: Algebra and Functions**

### **Algebraic Thinking as a Bridge to Functions Grades 3-6**

(Face-to-Face: *Summer 2013*; Online: *Fall 2013*)

This course will investigate the development of algebraic thinking to functional thinking using the multiple algebraic representations: table, graph, equation (with varying quantity represented), and problem situation. Participants will consider how to build upon students' use of algebraic properties when using operations to solve various problem situations to foment the use of the multiple representations to examine relationships between paired quantities. The topics include:

- Examining the behavior of arithmetic when quantities in the problem change;
- Generating tables of values that represent given rules and comparing them;
- Creating ordered pairs and graphing them in the coordinate plane to analyze relationships between the pairs of numbers; and

- Relating tables and graphs to simple equations.

This course does not have a pre-requisite, but prior coursework dealing with the role of algebraic thinking in the development of operations and algorithms will be helpful. This course sets a foundation for the course *Functions – Grades 6-12*.

## **Developing Proportional Reasoning Grades 4-7**

(Face-to-Face: *Summer 2013*; Online: *Fall 2013*)

This course will investigate the development of proportional reasoning concepts and skills beginning with multiplicative comparison and relating measurement units in grade 4 to using proportional relationships to solve multi-step problems in grade 7. Topics include:

- Analyzing multiplicative comparison multiplication and division problems, including measurement conversion;
- Revisiting the multiplicative rate of 10 in the base ten number system, including decimals;
- Developing the concepts and language of ratios and rate, including unit rates and percent; and
- Determining if two quantities are related proportionally, including representing proportional relationships with algebraic representations

This course does not have a pre-requisite, but connects nicely to ideas discussed in the course *Other Problem Situations: Multiplication and Division*. This course sets a foundation for the course *Functions – Grades 6-12*.

## **Functions Grades 6-12**

(Face-to-Face: *Summer 2013*; Online: *Fall 2013*)

This course uses as a required text the book Developing Essential Understanding of Expressions, Equations and Functions: Grades 6-8 from NCTM. The sessions will explore the five big ideas from the text and extend these ideas through high school. The five big ideas explored in this course are:

- Expressions
- Variables
- Equality
- Representing and analyzing functions
- Solving Equations

There is no pre-requisite for this course, but the course will align with two other courses in this series: *Algebraic Thinking as a Bridge to Functions – Grades 3-6* and *Developing Proportional Reasoning – Grades 4-7*.

### **CATEGORY: Chance, Data, and Statistics**

#### **Linear Measurement (course also listed under Geometry and Measurement) Grades K-3**

(Face-to-Face: *Summer 2013*; Online: *Fall 2013*)

This course will investigate the big ideas behind linear measurement, how children think about these ideas, the developmental trajectory of their thinking, and what this means instructionally.

{See full course description under the Geometry and Measurement CATEGORY.}

This course sets a foundation for all numerical data courses and all other measurement courses in this series.

#### **Numerical Data Grades 2-5**

(Face-to-Face: *Summer 2013*; Online: *Fall 2013*)

This course will focus on students' development of data displays and what the displays show and hide. Students need to collect data in repeated measurement contexts and use that information to make conclusions about the data. This class will use science as the foundation for data collection and for meaningful analysis. This course includes:

- Exploring the importance of repeated measurement contexts;
- Examining children's understanding of linear measurement and the impact of that understanding on data collection;
- Analyzing the different displays students make and what they show and hide;
- Identifying the different properties of displays and the importance of these properties; and
- Interpreting data displays by making connections to the goals of the data creation.

It is strongly recommended that participants complete the course *Linear Measurement – Grades K-3* before taking this course. Also, this course aligns with both *Data Modeling* courses for Grades 6-7.

**\*Data Modeling: Inventing Displays, Center, and Precision (Part I)**

**Grades 6-7**

(Face-to-Face: *Summer 2012*; Online: *Fall 2013*)

This course will develop statistical concepts for middle grades. It will explore representational competencies by examining what characteristics of data different displays make evident or even hide. Participants will then use data to determine a “best guess” of a real measurement using data, which leads to the development and understanding of measures of center by engaging an important mathematical practice—inventing an algorithm. Participants will then invent and explore methods for measuring the precision—the consistency—of a collection of measurements.

This course does not have a prerequisite course, but aligns with *Numerical Data – Grades 2-5*.

**\*Data Modeling: Chance, Modeling and Inference (Part II)**

**Grades 6-7**

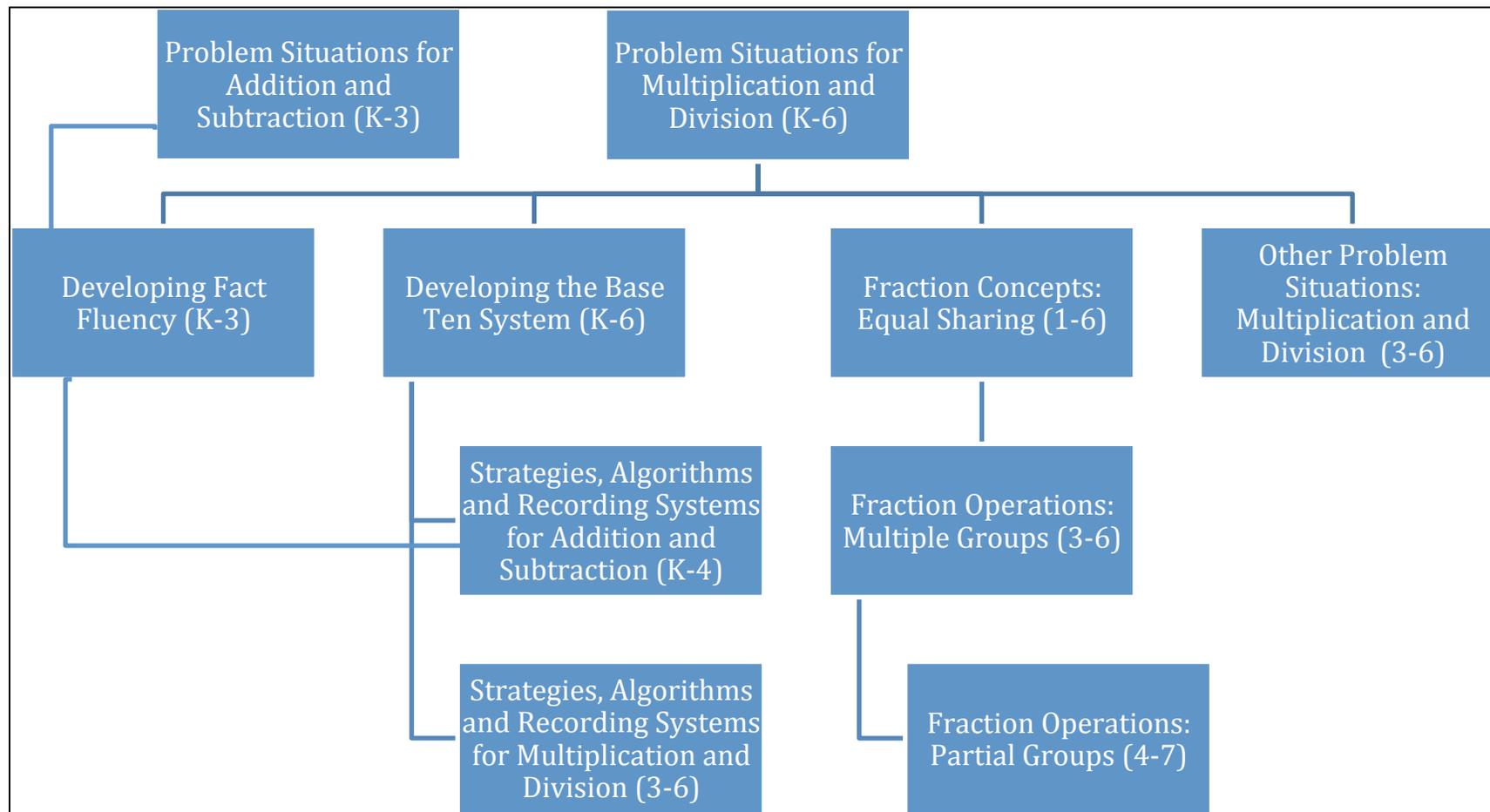
(Face-to-Face: *Summer 2013*; Online: *Fall 2013*)

This course will continue the development statistical concepts for middle grades. It builds upon the course *Data Modeling: Inventing Displays, Center and Precision*. Participants will use theoretical and empirical probabilities to estimate the chance of an event and consider the role of chance in data collection. They will use chance devices to create a model of a data collection scenario and examine sampling distributions to analyze the model’s fit and make inferences about claims.

This course requires the prerequisite course *Data Modeling: Inventing Displays, Center and Precision*.

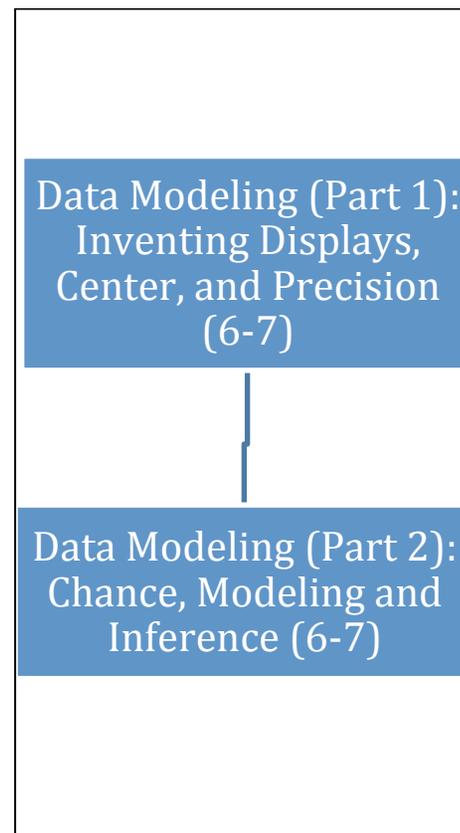
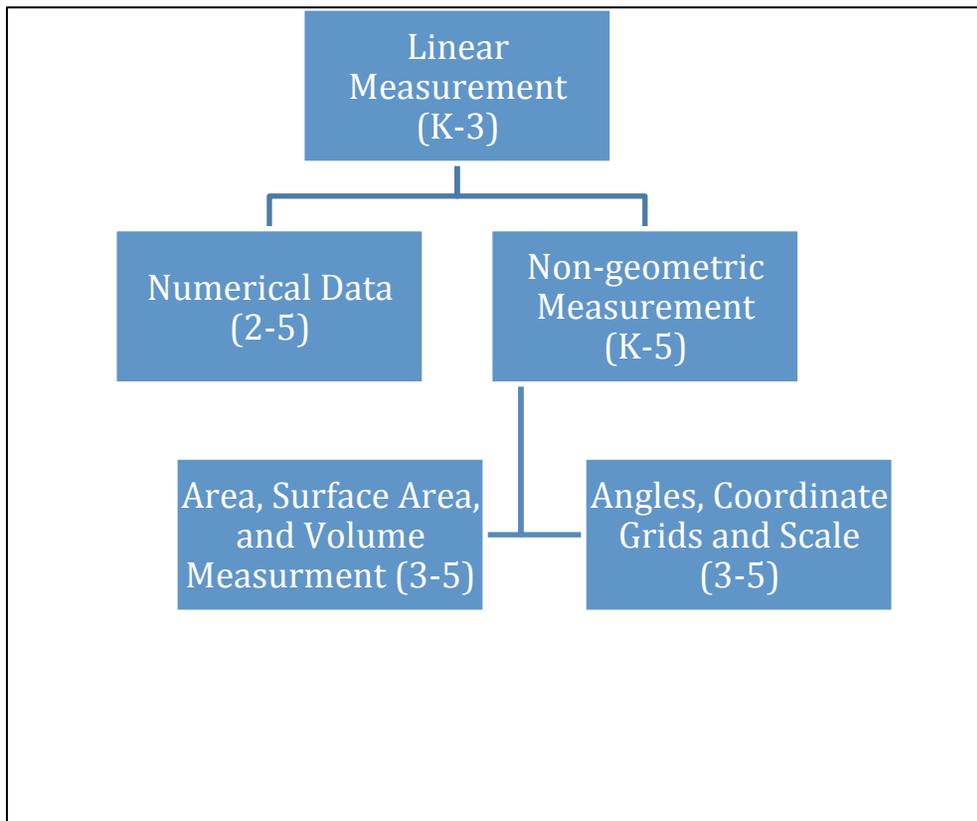
**\*[NOTE: At this time the *Data Modeling* courses can only be offered face-to-face by a specially trained group of facilitators through the co-ops and STEM centers. Therefore, these materials are not yet available as training materials for district use as for the other courses.]**

**Course Pathways: Based on Required or Recommended Prerequisites (Page 1 of 2)**

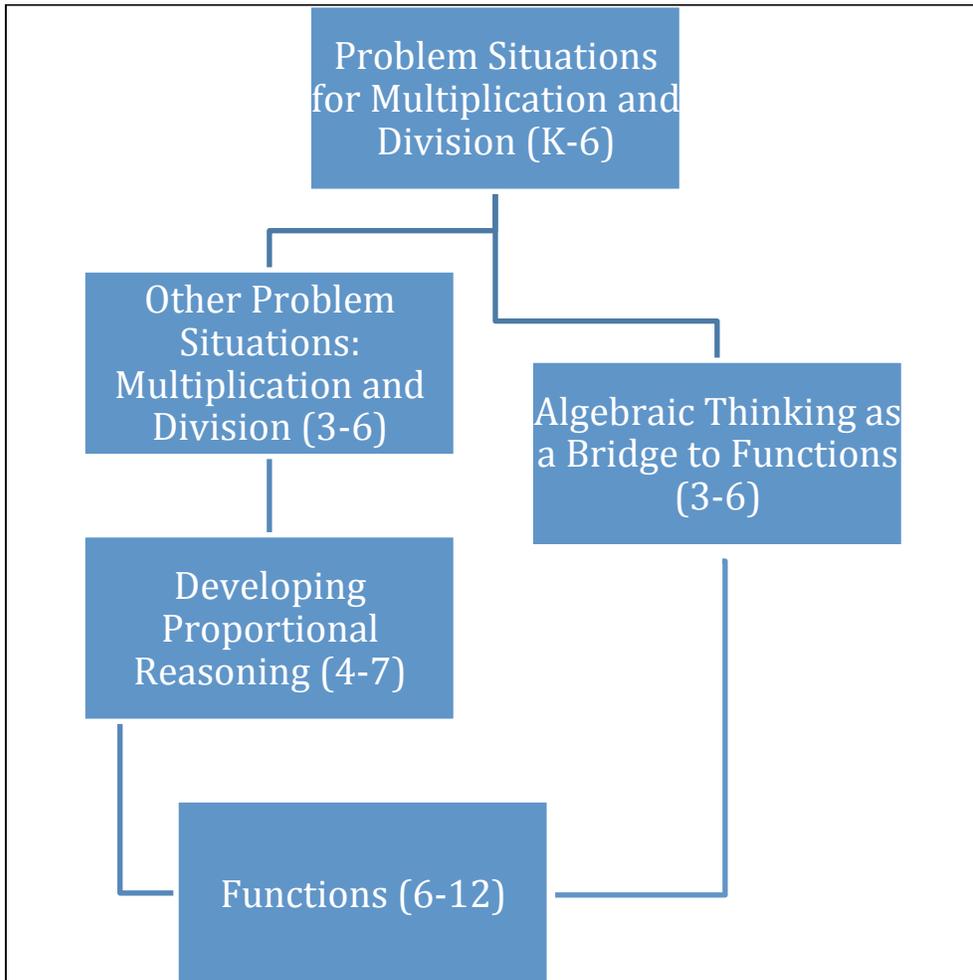


\*A teacher could proceed from Problem Situations for Addition and Subtraction to Developing Fact Fluency, and then to Strategies, Algorithms and Recording Systems for Addition and Subtraction, but she would want to pick up Developing the Base Ten System before taking Strategies, Algorithms and Recording Systems for Multiplication and Division.

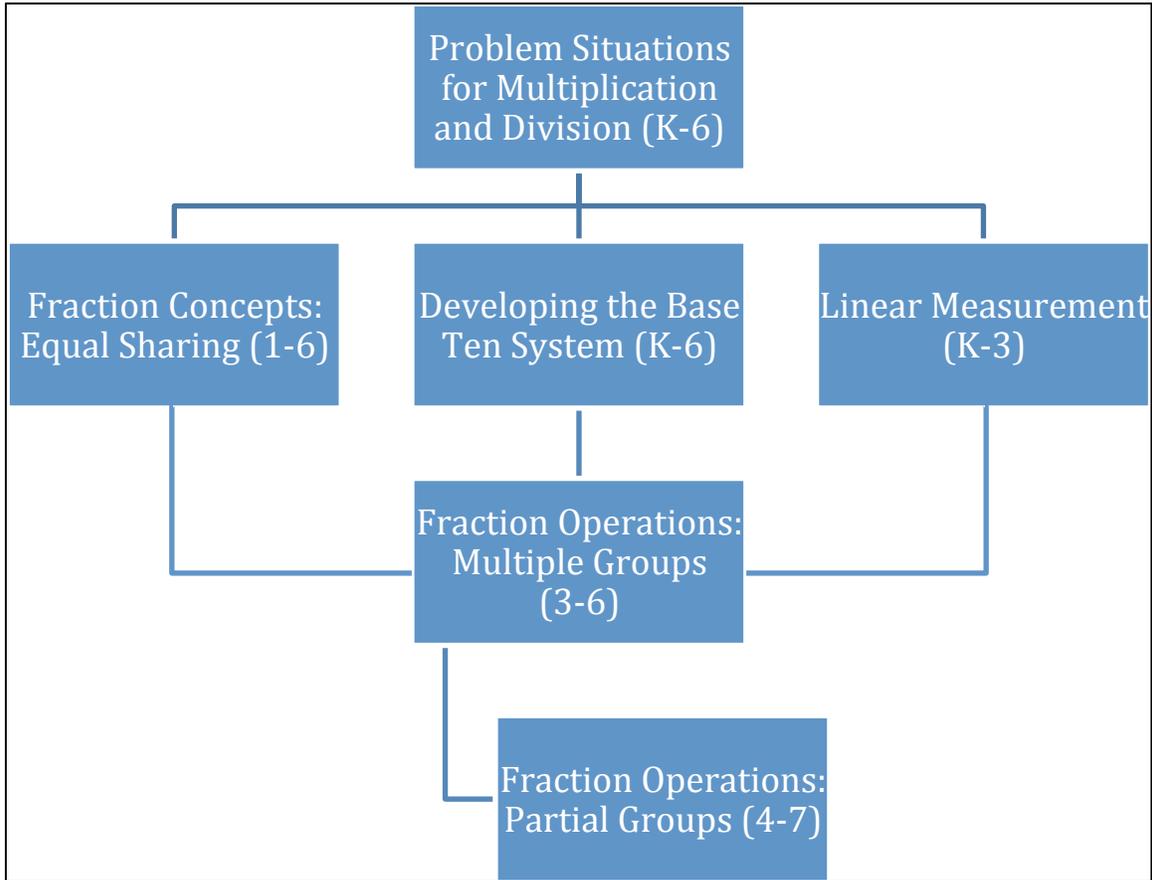
**Course Pathways: Based on Required or Recommended Prerequisites (Page 2 of 2)**



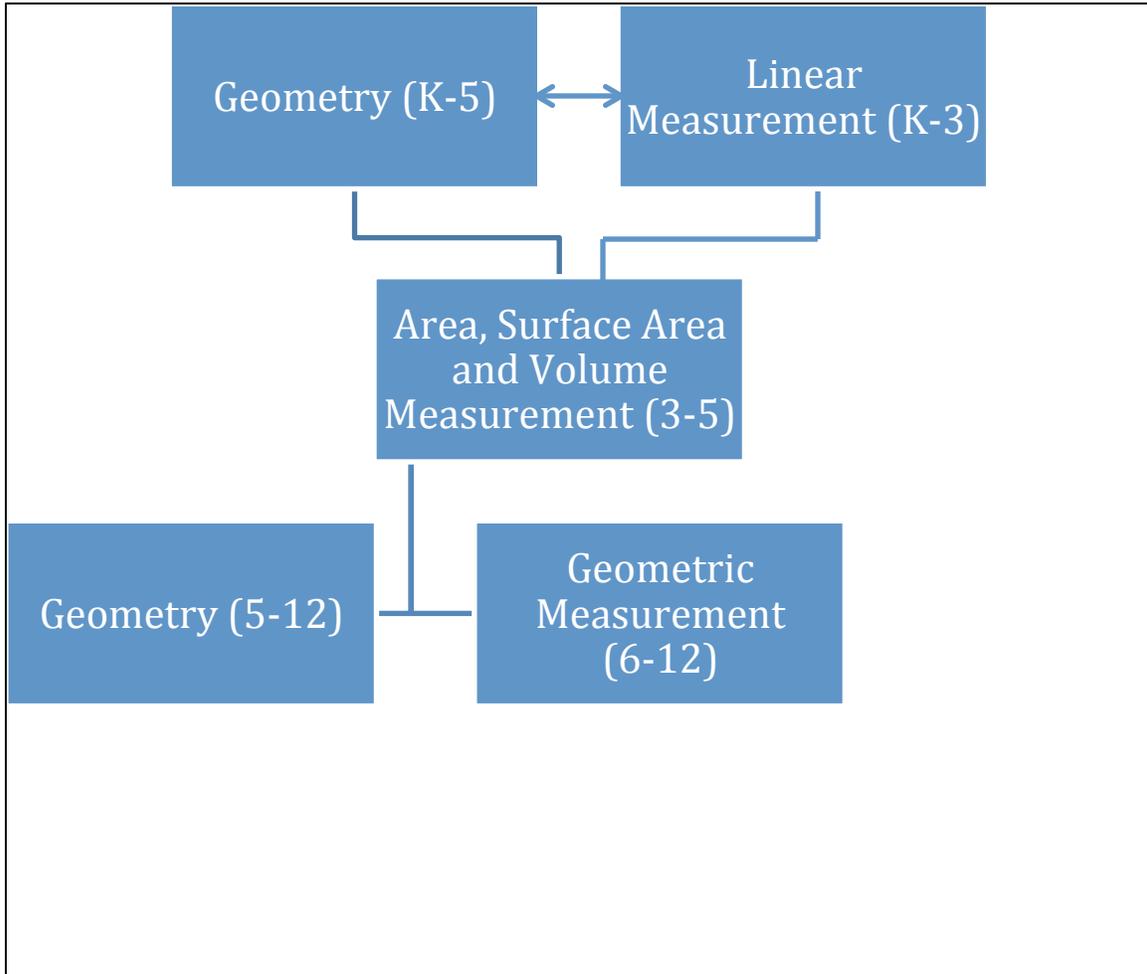
**Suggested Pathways for Vertical Articulation (Page 1 of 4)**



## Suggested Pathways for Vertical Articulation (Page 2 of 4)



**Suggested Pathways for Vertical Articulation (Page 3 of 4)**



## Suggested Pathways for Vertical Articulation (Page 4 of 4)

