Computer Science/Mathematics Crosswalk Committee Meeting
Monday August 12, 2013

On Monday, August 12, 2013 a statewide committee of Mathematics and Computer Science teachers and content specialist met in Birmingham to compare the content of the AP Computer Science A course and the Computer Science Principles Course (a new AP course that will be initiated in 2016-2017) with the Alabama Course of Study- Mathematics. The purpose of the comparison was to identify overlaps in content and to make recommendations to the State Department of Education regarding the option of awarding mathematics credit for completion of the Computer Science Courses.

The committee was asked to:

- **Review content of the CS Principles Course and the AP Computer Science A Course**
- **Review the high school mathematics courses as outlined in the Alabama Course of Study- Mathematics**
- **Review support documents to clarify content**
- **Map specific mathematics course of study objectives to content in the two CS courses**

In addition to mapping specific mathematics content standards, the committee also discussed the deep manner in which the content of the Computer Science Principles Course interweaves the Learning and Innovation Skills from *The Learning for 21st Century Framework*. These skills are what separate students who are prepared for increasingly complex life and work environments in today's world and those who are not. They include: Creativity and Innovation, Critical Thinking and Problem Solving, Communication, and Collaboration.

The committee also made the following specific recommendations regarding course sequence:

- **Computer Science Principles would be best taken either concurrently with Geometry or after the completion of Geometry. The College Board has not yet made any announcement about their suggestions for this course.**
- **AP Computer Science A would be best taken either concurrently with Algebra II or after completion of Algebra II or Algebra II with Trigonometry. The committee noted that the College Board suggests only Algebra I for this course.**

While both Computer Science Courses provide reinforcement and practical applications of many of the content standards in Algebra I and Geometry, the committee choose to focus instead on Algebra II, Algebra II with Trig, Discrete Mathematics, Mathematics Investigations, Pre-Calculus and Analytical Mathematics. The mapping summary documents are provided as a part of this report. The Course of Study Objectives are first mapped by course to the Learning Objectives in Computer Science Principles and to the topics in the AP Computer Science A course. Following that crosswalk, the COS objectives are listed with the Computer Science Content included.

As illustrated by the chart below, the greatest alignment for the Computer Science Principles Course (CSP) is with Discrete Math and Math Investigations; and for the Advanced Placement Computer Science A, the content is more evenly found in all of the courses.

Course	Number of Content Standards	Number of Standards Addressed in CSP	Number of Standards Addressed in AP CS
Algebra II	46	6	7
Algebra II/w Trig	50	6	7
Discrete Math	13	10	9
Math Investigations	12	7	2
Pre-Calculus	54	12	6
Analytical Math	18	2	4

Committee Members	
JEFF BAKER	Huntsville High School
JEFF GRAY	University of Alabama
KITTY MORGAN	A+ College Ready
JILL WESTERLUND	Hoover High School
KELLY ROUSE	Montgomery County Schools, retired
CAROL YARBROUGH	Alabama School of Fine Arts
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CINDY FREEMAN	SDE
DAWN MORRISON	SDE
MARY BOEHM	A+ College Ready
CAROL CRAWFORD	A+ College Ready

	Advanced Placement Computer Science A	Algebra II	Algebra II with Trig	Discrete Mathematics	Mathematical Investigations	Precalculus	Analytical Mathematics
Topic Outline	Description: Following is an outline of the major topics considered for the AP Computer Science A Exam. This outline is intended to define the scope of the course but not necessarily the sequence.						
	I. Object-Oriented Program Design The overall goal for designing a piece of software (a computer program) is to correctly solve the given problem. At the same time, this goal should encompass specifying and designing a program that is understandable, can be adapted to changing circumstances, and has the potential to be reused in whole or in part.			7a	2, 8		
A. Program design							
1	Read and understand a problem description, purpose, and goals.						
2	Apply data abstraction and encapsulation.						
3	Read and understand class specifications and relationships among the classes ("is-a," "has-a" relationships).						
4	Understand and implement a given class hierarchy.						
5	Identify reusable components from existing code using classes and class libraries.			1			
B. Class design		7,8	7,8			54 a and b, 45	

1	Design and implement a class.	33	33			19, 45	
2	Choose appropriate data representation and algorithms.	20	20			19	
3	Apply functional decomposition.					19	
4	Extend a given class using inheritance.			1		45	
	II. Program Implementation The overall goals of program implementation parallel those of program design. Classes that fill common needs should be built so that they can be reused easily in other programs. Object-oriented design is an important part of program implementation use computers to process information to gain insight and knowledge						
A. Implementation techniques				7a			
1	Methodology			2,3			
1.a	Object-oriented development					45	
1.b	Top-down development					12	
1.c	Encapsulation and information hiding						
1.d	Procedural abstraction			8		19	
B. Programming constructs				5			
1	Primitive types vs. objects						
2	Declaration						
2.a	Constant declarations						
2.b	Variable declarations						
2.c	Class declarations						
2.d	Interface declarations						
2.e	Method declarations						
2.f	Parameter declarations						
3	Console output (System.out.print/println)						
4	Control						13, 14
4.a	Methods						
4.b	Sequential						
4.c	Conditional						

4.d	Iteration						
4.e	Understand and evaluate recursive methods						
C. Java library classes	(included in the AP Java subset)	37, 38	41, 42	1		50, 51,54	
	III. Program Analysis The analysis of programs includes examining and testing programs to determine whether they correctly meet their specifications. It also includes the analysis of programs or algorithms in order to understand their time and space requirements when applied to different data sets. analyze how computing affects communication, interaction, and cognition						
A. Testing							
1	Test classes and libraries in isolation.						
2	Identify boundary cases and generate appropriate test data.						
3	Perform integration testing.						
B. Debugging							
1	Categorize errors: compile-time, run-time, logic.						
2	Identify and correct errors.						
3	Employ techniques such as using a debugger, adding extra output statements, or hand-tracing code.						
C. Understand and modify existing code							
D. Extend existing code using inheritance						45	
E. Understand error							

handling							
1.	Understand runtime exceptions.						
F. Reason about programs						45	
1.	Pre- and post-conditions						
2.	Assertions						
G. Analysis of algorithms		7,8	7,8	9		12	
1.	Informal comparisons of running times	27	27				
2.	Exact calculation of statement execution counts					24	
H. Numerical representations and limits				4			
1	Representations of numbers in different bases			4			
2	Limitations of finite representations (e.g., integer bounds, imprecision of floating-point representations, and round-off error)						
	IV. Standard Data Structures Data structures are used to represent information within a program. Abstraction is an important theme in the development and application of data structures.						
A. Simple data types (int, boolean, double)							
B. Classes							
C. Lists							
D. Arrays		7,8	7,8	11		45	6,7
	V. Standard Algorithms Standard algorithms serve as examples of good solutions to standard problems. Many are intertwined with						

	standard data structures. These algorithms provide examples for analysis of program efficiency.						
A. Operations on data structures previously listed							
1	Traversals						
2	Insertions						
3	Deletions						
B. Searching							
1.	Sequential						
2.	Binary						
C. Sorting							
1	Selection						
2	Insertion						
3	Mergesort						
	VI. Computing in Context An awareness of the ethical and social implications of computing systems is necessary for the study of computer science. These topics need not be addressed in detail but should be considered throughout the course.				11		
A. System reliability							
B. Privacy							
C. Legal issues and intellectual property							
D. Social and ethical ramifications of computer use							

ALGEBRA II

Algebra II is a terminating course designed to extend students' algebraic knowledge and skills beyond Algebra I. Students are encouraged to solve problems using a variety of methods that promote the development of improved communication skills and foster a deeper understanding of mathematics. To help students appreciate the power of algebra, application-based problems are incorporated throughout the course. The use of appropriate technology is also encouraged for numerical and graphical investigations.

In contrast to the Algebra II With Trigonometry course, Algebra II does not meet the graduation requirements for the Alabama High School Diploma with Advanced Academic Endorsement due to the fact that it does not contain trigonometry content. Algebra II With Trigonometry or Algebra II is required to complete the graduation requirements for the Alabama High School Diploma. This course does not provide sufficient background to prepare students to pursue higher-level mathematics courses. The prerequisites for Algebra II are Algebra I and Geometry.
Students will:

NUMBER AND QUANTITY

The Complex Number System

Perform arithmetic operations with complex numbers.

1. Know there is a complex number i such that $i^2 = -1$, and every complex number has the form $a + bi$ with a and b real. [N-CN1]

2. Use the relation $i^2 = -1$ and the commutative, associative, and distributive properties to add, subtract, and multiply complex numbers. [N-CN2]

3. (+) Find the conjugate of a complex number; use conjugates to find moduli and quotients of complex numbers. [N-CN3]

Use complex numbers in polynomial identities and equations. ***(Polynomials with real coefficients.)***

4. Solve quadratic equations with real coefficients that have complex solutions. [N-CN7]

5. (+) Extend polynomial identities to the complex numbers. [N-CN8]
   Example: Rewrite $x^2 + 4$ as $(x + 2i)(x - 2i)$.

6. (+) Know the Fundamental Theorem of Algebra; show that it is true for quadratic polynomials. [N-CN9]

Vector and Matrix Quantities

Perform operations on matrices and use matrices in applications.

7. (+) Use matrices to represent and manipulate data, e.g., to represent payoffs or incidence relationships in a network. (*Use technology to approximate roots.)* [N-VM6]

Object-Oriented Program Design. B. Class Design
Program Implementation G. Analysis of algorithms
Standard Data Structures. D. Arrays

8. (+) Multiply matrices by scalars to produce new matrices, e.g., as when all of the payoffs in a game are doubled. [N-VM7]

Object-Oriented Program Design. B. Class Design
Program Implementation G. Analysis of algorithms
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9. (+) Add, subtract, and multiply matrices of appropriate dimensions. [N-VM8]

10. (+) Understand that, unlike multiplication of numbers, matrix multiplication for square matrices is not a commutative operation, but still satisfies the associative and distributive properties. [N-VM9]

11. (+) Understand that the zero and identity matrices play a role in matrix addition and multiplication similar to the role of 0 and 1 in the real numbers. The determinant of a square matrix is nonzero if and only if the matrix has a multiplicative inverse. [N-VM10]

ALGEBRA

Seeing Structure in Expressions

Interpret the structure of expressions. (*Polynomial and rational.)*

12. Interpret expressions that represent a quantity in terms of its context.* [A-SSE1]
    a. Interpret parts of an expression such as terms, factors, and coefficients. [A-SSE1a]
    b. Interpret complicated expressions by viewing one or more of their parts as a single entity. [A-SSE1b]
       Example: Interpret $P(1+r)^n$ as the product of P and a factor not depending on P.

13. Use the structure of an expression to identify ways to rewrite it. [A-SSE2]
    Example: See $x^4 - y^4$ as $(x^2)^2 - (y^2)^2$, thus recognizing it as a difference of squares that can be factored as $(x^2 - y^2)(x^2 + y^2)$.

Write expressions in equivalent forms to solve problems.

14. Derive the formula for the sum of a finite geometric series (when the common ratio is not 1), and use the formula to solve problems.* [A-SSE4]
    Example: Calculate mortgage payments.

Arithmetic With Polynomials and Rational Expressions

Perform arithmetic operations on polynomials. *(Beyond quadratic.)*

15. Understand that polynomials form a system analogous to the integers; namely, they are closed under the operations of addition, subtraction, and multiplication; add, subtract, and multiply polynomials. [A-APR1]

Understand the relationship between zeros and factors of polynomials.

16. Know and apply the Remainder Theorem: For a polynomial $p(x)$ and a number a, the remainder on division by $x - a$ is $p(a)$, so $p(a) = 0$ if and only if $(x - a)$ is a factor of $p(x)$. [A-APR2]

17. Identify zeros of polynomials when suitable factorizations are available, and use the zeros to construct a rough graph of the function defined by the polynomial. [A-APR3]

Use polynomial identities to solve problems.

18. Prove polynomial identities and use them to describe numerical relationships. [A-APR4]
    Example: The polynomial identity $(x^2 + y^2)^2 = (x^2 - y^2)^2 + (2xy)^2$ can be used to generate Pythagorean triples.

Rewrite rational expressions. ***(Linear and quadratic denominators.)***

19. Rewrite simple rational expressions in different forms; write $a(x)/b(x)$ in the form $q(x) + r(x)/b(x)$, where $a(x)$, $b(x)$, $q(x)$, and $r(x)$ are polynomials with the degree of $r(x)$ less than the degree of $b(x)$, using inspection, long division, or for the more complicated examples, a computer algebra system. [A-APR6]

Creating Equations*

Create equations that describe numbers or relationships. ***(Equations using all available types of expressions, including simple root functions.)***

20. Create equations and inequalities in one variable and use them to solve problems. *Include equations arising from linear and quadratic functions, and simple rational and exponential functions.* [A-CED1]

Object-Oriented Program Design. B. Class Design. 2. Choose appropriate data representation and algorithms.

21. Create equations in two or more variables to represent relationships between quantities; graph equations on coordinate axes with labels and scales. [A-CED2]

22. Represent constraints by equations or inequalities, and by systems of equations and/or inequalities, and interpret solutions as viable or nonviable options in a modeling context. [A-CED3]
    Example: Represent inequalities describing nutritional and cost constraints on combinations of different foods.

23. Rearrange formulas to highlight a quantity of interest, using the same reasoning as in solving equations. [A-CED4]
    Example: Rearrange Ohm's law $V = IR$ to highlight resistance R.

Reasoning With Equations and Inequalities

Understand solving equations as a process of reasoning and explain the reasoning. ***(Simple rational and radical.)***

24. Solve simple rational and radical equations in one variable, and give examples showing how extraneous solutions may arise. [A-REI2]

Solve equations and inequalities in one variable.

25. Recognize when the quadratic formula gives complex solutions, and write them as $a \pm bi$ for real numbers a and b. [A-REI4b] **Solve systems of equations.**

26. (+) Find the inverse of a matrix if it exists and use it to solve systems of linear equations (using technology for matrices of dimension 3 × 3 or greater). [A-REI9]

Represent and solve equations and inequalities graphically. ***(Combine polynomial, rational, radical, absolute value, and exponential functions.)***

27. Explain why the x-coordinates of the points where the graphs of the equations $y = f(x)$ and $y = g(x)$ intersect are the solutions of the equation $f(x) = g(x)$; find the solutions approximately, e.g., using technology to graph the functions, make tables of values, or find successive approximations. Include cases where $f(x)$ and/or $g(x)$ are linear, polynomial, rational, absolute value, exponential, and logarithmic functions.* [A-REI11]

Program Analysis G. Analysis of algorithms 1. Informal comparisons of running times

Conic Sections

Understand the graphs and equations of conic sections. ***(Emphasize understanding graphs and equations of circles and parabolas.)***

28. Create graphs of conic sections, including parabolas, hyperbolas, ellipses, circles, and degenerate conics, from second-degree equations.

    Example: Graph $x^2 - 6x + y^2 - 12y + 41 = 0$ or $y^2 - 4x + 2y + 5 = 0$.

    a. Formulate equations of conic sections from their determining characteristics.

        Example: Write the equation of an ellipse with center (5, -3), a horizontal major axis of length 10, and a minor axis of length 4.

        Answer: $\frac{(x-5)^2}{25} + \frac{(y+3)^2}{4} = 1$.

FUNCTIONS

Interpreting Functions

Interpret functions that arise in applications in terms of the context. ***(Emphasize selection of appropriate models.)***

29. Relate the domain of a function to its graph and, where applicable, to the quantitative relationship it describes.* [F-IF5]

    Example: If the function $h(n)$ gives the number of person-hours it takes to assemble n engines in a factory, then the positive integers would be an appropriate domain for the function.

Analyze functions using different representations. ***(Focus on using key features to guide selection of appropriate type of model function.)***

30. Graph functions expressed symbolically, and show key features of the graph, by hand in simple cases and using technology for more complicated cases.* [F-IF7]

a. Graph square root, cube root, and piecewise-defined functions, including step functions and absolute value functions. [F-IF7b]
b. Graph polynomial functions, identifying zeros when suitable factorizations are available, and showing end behavior. [F-IF7c]
c. Graph exponential and logarithmic functions, showing intercepts and end behavior, and trigonometric functions, showing period, midline, and amplitude. [F-IF7e]

31. Write a function defined by an expression in different but equivalent forms to reveal and explain different properties of the function. [F-IF8]

32. Compare properties of two functions each represented in a different way (algebraically, graphically, numerically in tables, or by verbal descriptions). [F-IF9]
    Example: Given a graph of one quadratic function and an algebraic expression for another, say which has the larger maximum.

Building Functions

Build a function that models a relationship between two quantities. ***(Include all types of functions studied.)***

33. Write a function that describes a relationship between two quantities.* [F-BF1]
    a. Combine standard function types using arithmetic operations. [F-BF1b]
        Example: Build a function that models the temperature of a cooling body by adding a constant function to a decaying exponential, and relate these functions to the model.

Build new functions from existing functions. ***(Include simple radical, rational, and exponential functions; emphasize common effect of each transformation across function types.)***

Object-Oriented Program Design. B. Class Design 1. Design and Implement a class

34. Identify the effect on the graph of replacing $f(x)$ by $f(x) + k$, $k\,f(x)$, $f(kx)$, and $f(x + k)$ for specific values of k (both positive and negative); find the value of k given the graphs. Experiment with cases and illustrate an explanation of the effects on the graph using technology. Include recognizing even and odd functions from their graphs and algebraic expressions for them. [F-BF3]

35. Find inverse functions. [F-BF4]
    a. Solve an equation of the form $f(x) = c$ for a simple function f that has an inverse, and write an expression for the inverse. [F-BF4a]
        Example: $f(x) =2x^3$ or $f(x) = (x+1)/(x-1)$ for $x \neq 1$.

Linear, Quadratic, and Exponential Models*

Construct and compare linear, quadratic, and exponential models and solve problems. ***(Logarithms as solutions for exponentials.)***

36. For exponential models, express as a logarithm the solution to $ab^{ct} = d$ where a, c, and d are numbers, and the base b is 2, 10, or e; evaluate the logarithm using technology. [F-LE4]

STATISTICS AND PROBABILITY

Using Probability to Make Decisions

Use probability to evaluate outcomes of decisions. ***(Include more complex situations.)***

37. (+) Use probabilities to make fair decisions (e.g., drawing by lots, using a random number generator). [S-MD6]

Program Implementation. C. Java Library Classes

38. (+) Analyze decisions and strategies using probability concepts (e.g., product testing, medical testing, pulling a hockey goalie at the end of a game). [S-MD7]

Program Implementation. C. Java Library Classes

Conditional Probability and the Rules of Probability

Understand independence and conditional probability and use them to interpret data.
(Link to data from simulations or experiments.)

39. Describe events as subsets of a sample space (the set of outcomes), using characteristics (or categories) of the outcomes, or as unions, intersections, or complements of other events ("or," "and," "not"). [S-CP1]

40. Understand the conditional probability of A given B as $P(A \text{ and } B)/P(B)$, and interpret independence of A and B as saying that the conditional probability of A given B is the same as the probability of A, and the conditional probability of B given A is the same as the probability of B. [S-CP3]

41. Construct and interpret two-way frequency tables of data when two categories are associated with each object being classified. Use the two-way table as a sample space to decide if events are independent and to approximate conditional probabilities. [S-CP4]

    Example: Collect data from a random sample of students in your school on their favorite subject among mathematics, science, and English. Estimate the probability that a randomly selected student from your school will favor science given that the student is in tenth grade. Do the same for other subjects and compare the results.

42. Recognize and explain the concepts of conditional probability and independence in everyday language and everyday situations. [S-CP5]

    Example: Compare the chance of having lung cancer if you are a smoker with the chance of being a smoker if you have lung cancer.

Use the rules of probability to compute probabilities of compound events in a uniform probability model.

43. Find the conditional probability of A given B as the fraction of B's outcomes that also belong to A, and interpret the answer in terms of the model. [S-CP6]

44. Apply the Addition Rule, $P(A \text{ or } B) = P(A) + P(B) - P(A \text{ and } B)$, and interpret the answer in terms of the model. [S-CP7]

45. (+) Apply the general Multiplication Rule in a uniform probability model, $P(A \text{ and } B) = P(A)P(B|A) = P(B)P(A|B)$, and interpret the answer in terms of the model. [S-CP8]

46. (+) Use permutations and combinations to compute probabilities of compound events and solve problems. [S-CP9]

ALGEBRA II WITH TRIGONOMETRY

Algebra II With Trigonometry is a course designed to extend students' knowledge of Algebra I with additional algebraic and trigonometric content. Mastery of the content standards for this course is necessary for student success in higher-level mathematics. The use of appropriate technology is encouraged for numerical and graphical investigations that enhance analytical comprehension.

Algebra II With Trigonometry is required for all students pursuing the Alabama High School Diploma with Advanced Academic Endorsement. Prerequisites for this course are Algebra I and Geometry. If a student chooses to take the Algebraic Connections course, it must be taken prior to the Algebra II With Trigonometry course.
Students will:

NUMBER AND QUANTITY

The Complex Number System

Perform arithmetic operations with complex numbers.

1. Know there is a complex number i such that $i^2 = -1$, and every complex number has the form $a + bi$ with a and b real. [N-CN1]

2. Use the relation $i^2 = -1$ and the commutative, associative, and distributive properties to add, subtract, and multiply complex numbers. [N-CN2]

3. (+) Find the conjugate of a complex number; use conjugates to find moduli and quotients of complex numbers. [N-CN3]

Use complex numbers in polynomial identities and equations. ***(Polynomials with real coefficients.)***

4. Solve quadratic equations with real coefficients that have complex solutions. [N-CN7]

5. (+) Extend polynomial identities to the complex numbers.
   Example: Rewrite $x^2 + 4$ as $(x + 2i)(x - 2i)$. [N-CN8]

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7. (+) Use matrices to represent and manipulate data, e.g., to represent payoffs or incidence relationships in a network. (*Use technology to approximate roots.)* [N-VM6]

Object-Oriented Program Design. B. Class Design
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Object-Oriented Program Design. B. Class Design 2. Choose appropriate data representation and algorithms.

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Understand solving equations as a process of reasoning, and explain the reasoning. ***(Simple rational and radical.)***

24. Solve simple rational and radical equations in one variable, and give examples showing how extraneous solutions may arise. [A-REI2]

Solve equations and inequalities in one variable.

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Solve systems of equations.

26. (+) Find the inverse of a matrix if it exists and use it to solve systems of linear equations (using technology for matrices of dimension 3 × 3 or greater). [A-REI9]

Represent and solve equations and inequalities graphically. ***(Combine polynomial, rational, radical, absolute value, and exponential functions.)***

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28. Create graphs of conic sections, including parabolas, hyperbolas, ellipses, circles, and degenerate conics, from second-degree equations.

    Example: Graph $x^2 - 6x + y^2 - 12y + 41 = 0$ or $y^2 - 4x + 2y + 5 = 0$.

    a. Formulate equations of conic sections from their determining characteristics.

       Example: Write the equation of an ellipse with center (5, -3), a horizontal major axis of length 10, and a minor axis of length 4.

       Answer: $\frac{(x-5)^2}{25} + \frac{(y+3)^2}{4} = 1.$

FUNCTIONS

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Interpret functions that arise in applications in terms of the context. ***(Emphasize selection of appropriate models.)***

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    Example: If the function $h(n)$ gives the number of person-hours it takes to assemble n engines in a factory, then the positive integers would be an appropriate domain for the function.

Analyze functions using different representations. ***(Focus on using key features to guide selection of appropriate type of model function.)***

30. Graph functions expressed symbolically, and show key features of the graph, by hand in simple cases and using technology for more complicated cases.* [F-IF7]
    a. Graph square root, cube root, and piecewise-defined functions, including step functions and absolute value functions. [F-IF7b]
    b. Graph polynomial functions, identifying zeros when suitable factorizations are available, and showing end behavior. [F-IF7c]
    c. Graph exponential and logarithmic functions, showing intercepts and end behavior, and trigonometric functions, showing period, midline, and amplitude. [F-IF7e]

31. Write a function defined by an expression in different but equivalent forms to reveal and explain different properties of the function. [F-IF8]

32. Compare properties of two functions each represented in a different way (algebraically, graphically, numerically in tables, or by verbal descriptions). [F-IF9]
    Example: Given a graph of one quadratic function and an algebraic expression for another, say which has the larger maximum.

Building Functions

Build a function that models a relationship between two quantities. ***(Include all types of functions studied.)***

33. Write a function that describes a relationship between two quantities.* [F-BF1]
    a. Combine standard function types using arithmetic operations. [F-BF1b]
       Example: Build a function that models the temperature of a cooling body by adding a constant function to a decaying exponential, and relate these functions to the model.

Build new functions from existing functions. ***(Include simple radical, rational, and exponential functions; emphasize common effect of each transformation across function types.)***

Object-Oriented Program Design. B. Class Design 1. Design and implement a class

34. Identify the effect on the graph of replacing $f(x)$ by $f(x) + k$, $k\,f(x)$, $f(kx)$, and $f(x + k)$ for specific values of k (both positive and negative); find the value of k given the graphs. Experiment with cases and illustrate an explanation of the effects on the graph using technology. Include recognizing even and odd functions from their graphs and algebraic expressions for them. [F-BF3]

35. Find inverse functions. [F-BF4]
    a. Solve an equation of the form $f(x) = c$ for a simple function f that has an inverse, and write an expression for the inverse. [F-BF4a]
       Example: $f(x) = 2x^3$ or $f(x) = (x+1)/(x-1)$ for $x \neq 1$.

Linear, Quadratic, and Exponential Models*

Construct and compare linear, quadratic, and exponential models and solve problems. ***(Logarithms as solutions for exponentials.)***

36. For exponential models, express as a logarithm the solution to $ab^{ct} = d$ where a, c, and d are numbers, and the base b is 2, 10, or e; evaluate the logarithm using technology. [F-LE4]

Trigonometric Functions

Extend the domain of trigonometric functions using the unit circle.

37. Understand radian measure of an angle as the length of the arc on the unit circle subtended by the angle. [F-TF1]

38. Explain how the unit circle in the coordinate plane enables the extension of trigonometric functions to all real numbers, interpreted as radian measures of angles traversed counterclockwise around the unit circle. [F-TF2]

39. Define the six trigonometric functions using ratios of the sides of a right triangle, coordinates on the unit circle, and the reciprocal of other functions.

Model periodic phenomena with trigonometric functions.

40. Choose trigonometric functions to model periodic phenomena with specified amplitude, frequency, and midline.* [F-TF5]

STATISTICS AND PROBABILITY

Using Probability to Make Decisions

Use probability to evaluate outcomes of decisions. (*Include more complex situations.*)

41. (+) Use probabilities to make fair decisions (e.g., drawing by lots, using a random number generator). [S-MD6]

Object-Oriented Program Design C. Java library classes.

42. (+) Analyze decisions and strategies using probability concepts (e.g., product testing, medical testing, pulling a hockey goalie at the end of a game). [S-MD7]

Object-Oriented Program Design C. Java library classes.

Conditional Probability and the Rules of Probability

Understand independence and conditional probability and use them to interpret data. ***(Link to data from simulations or experiments.)***

43. Describe events as subsets of a sample space (the set of outcomes), using characteristics (or categories) of the outcomes, or as unions, intersections, or complements of other events ("or," "and," "not"). [S-CP1]

44. Understand the conditional probability of *A* given *B* as *P*(*A* and *B*)/*P*(*B*), and interpret independence of *A* and *B* as saying that the conditional probability of *A* given *B* is the same as the probability of *A*, and the conditional probability of *B* given *A* is the same as the probability of *B*. [S-CP3]

45. Construct and interpret two-way frequency tables of data when two categories are associated with each object being classified. Use the two-way table as a sample space to decide if events are independent and to approximate conditional probabilities. [S-CP4]
    Example: Collect data from a random sample of students in your school on their favorite subject among mathematics, science, and English. Estimate the probability that a randomly selected student from your school will favor science given that the student is in tenth grade. Do the same for other subjects and compare the results.

46. Recognize and explain the concepts of conditional probability and independence in everyday language and everyday situations. [S-CP5]
    Example: Compare the chance of having lung cancer if you are a smoker with the chance of being a smoker if you have lung cancer.

Use the rules of probability to compute probabilities of compound events in a uniform probability model.

47. Find the conditional probability of *A* given *B* as the fraction of *B*'s outcomes that also belong to *A*, and interpret the answer in terms of the model. [S-CP6]

48. Apply the Addition Rule, $P(A \text{ or } B) = P(A) + P(B) - P(A \text{ and } B)$, and interpret the answer in terms of the model. [S-CP7]

49. (+) Apply the general Multiplication Rule in a uniform probability model, $P(A \text{ and } B) = P(A)P(B|A) = P(B)P(A|B)$, and interpret the answer in terms of the model. [S-CP8]

50. (+) Use permutations and combinations to compute probabilities of compound events and solve problems. [S-CP9]

DISCRETE MATHEMATICS

Discrete Mathematics is a course designed for students who have successfully completed the Algebra II With Trigonometry course and who choose not to continue mathematics study in the Precalculus or Analytical Mathematics courses. This course may be offered as an elective for students who have completed the four mathematics requirements for graduation.

Discrete Mathematics expands upon the topics of matrices, combinational reasoning, counting techniques, algorithms, sequences, series, and their applications. Students are expected to work in both individual and group settings to apply problem-solving strategies and to incorporate technological tools that extend beyond traditional instructional practices. The prerequisites for this course are Algebra I, Geometry, and Algebra II With Trigonometry.

Students will:

NUMBER AND QUANTITY

1. Analyze topics from elementary number theory, including perfect numbers and prime numbers, to determine properties of integers.

Object Oriented Program Design- A. Program Design. 5 Identify reusable components from existing code using classes and class libraries.

Object Oriented Program Design- B. Class Design. 4 Extend a given class using inheritance.

Object Oriented Program Design C. Java Library

2. Determine characteristics of sequences, including the Fibonacci sequence, the triangular numbers, and pentagonal numbers.

   Example: Write a sequence of the first 10 triangular numbers and hypothesize a formula to find the n^{th} triangular number.

Program Implementation. A. Implementation techniques. 1. Methodology

3. Use the recursive process and difference equations to create fractals, population growth models, sequences, series, and compound interest models.

Program Implementation. A. Implementation techniques. 1. Methodology

4. Convert between base ten and other bases.

Program Analysis. H. Numerical representations and limits

Program Analysis. H. Numerical representations and limits. 1. Representation of number in different bases.

ALGEBRA

5. Determine results of operations upon 3 × 3 and larger matrices, including matrix addition and multiplication of a matrix by a matrix, vector, or scalar.

Programming Constructs.

6. Analyze determinants and inverses of 2 × 2, 3 × 3, and larger matrices to determine the nature of the solution set of the corresponding system of equations, including solving systems of equations in three variables by echelon row reduction and matrix inverse.

7. Solve problems through investigation and application of existence and nonexistence of Euler paths, Euler circuits, Hamilton paths, and Hamilton circuits.

   Example: Show why a 5 × 5 grid has no Hamilton circuit.

   a. Develop optimal solutions of application-based problems using existing and student-created algorithms

Object-Oriented Program Design

Program implementation. A. Implementation techniques

8. Apply algorithms, including Kruskal's and Prim's, relating to minimum weight spanning trees, networks, flows, and Steiner trees.

   a. Use shortest path techniques to find optimal shipping routes.

Program Implementation A. Implementation techniques. 1.D. Procedural abstraction

9. Determine a minimum project time using algorithms to schedule tasks in order, including critical path analysis, the list-processing algorithm, and student-created algorithms.

Program Analysis G. Analysis of algorithms Program Analysis G. Analysis of algorithms 1. Informal comparisons of running times

DISCRETE MATHEMATICS

GEOMETRY

10. Use vertex-coloring techniques and matching techniques to solve application-based problems.

    Example: Use graph-coloring techniques to color a map of the western states of the United States so no adjacent states are the same color, including determining the minimum number of colors needed and why no fewer colors may be used.

11. Solve application-based logic problems using Venn diagrams, truth tables, and matrices.

Standard Data Structures. . A. Simple Data Types. D. Arrays

STATISTICS AND PROBABILITY

12. Use combinatorial reasoning and counting techniques to solve application-based problems.

    Example: Determine the probability of a safe opening on the first attempt given the combination uses the digits 2, 4, 6, and 8 with the order unknown.

    Answer: The probability of the safe opening on the first attempt is $\frac{1}{24}$.

13. Analyze election data to compare election methods and voting apportionment, including determining strength within specific groups.

MATHEMATICAL INVESTIGATIONS

Mathematical Investigations is a course designed for students who have successfully completed the Algebra II With Trigonometry course and who choose not to continue mathematics study in the Precalculus or Analytical Mathematics courses. This course may be offered as an elective for students who have completed the four mathematics requirements for graduation.

Mathematical Investigations is intended to extend students' knowledge of mathematical development. Beginning with ancient numeration systems, students explore relationships between mathematics and nature, music, art, and architecture as well as the contributions of well-known mathematicians. It extends the scope of prerequisite courses, integrating topics with an emphasis on application-based problem solving. The wide range of topics and applied problems may lend itself to organizing the content into thematic units. The prerequisites for this course are Algebra I, Geometry, and Algebra II With Trigonometry.

Students will:

NUMBER AND QUANTITY

1. Critique ancient numeration systems and applications, including astronomy and the development and use of money and calendars.
   a. Determine relationships among mathematical achievements of ancient peoples, including the Sumerians, Babylonians, Egyptians, Mesopotamians, Chinese, Aztecs, and Incas.
   b. Explain origins of the Hindu-Arabic numeration system.
      Example: Perform addition and subtraction in both the Hindu-Arabic and the Roman numeration systems to compare place value and place holders.

2. Analyze mathematical relationships in music to interpret frequencies of musical notes and to compare mathematical structures of various musical instruments.
   Examples: Compare frequencies of notes exactly one octave apart on the musical scale; using frequencies and wave patterns of middle C, E above middle C, and G above middle C to explain why the C major chord is harmonious.

Object-oriented Program Design

   a. Determine lengths of strings necessary to produce harmonic tones as in Pythagorean tuning.

3. Use special numbers, including e, i, π, and the golden ratio, to solve application-based problems.
   a. Identify transcendental numbers.
      Example: Calculate e to ten decimal places using a summation with $\frac{1}{n!}$.

4. Explain the development and uses of sets of numbers, including complex, real, rational, irrational, integer, whole, and natural numbers.

a. Analyze contributions to the number system by well-known mathematicians, including Archimedes, John Napier, René Descartes, Sir Isaac Newton, Johann Carl Friedrich Gauss, and Julius Wilhelm Richard Dedekind.

Example: Plot solutions to the polynomial equation, $x^2 - 6x + 11 = 0$, on the Gaussian plan

ALGEBRA

5. Identify beginnings of algebraic symbolism and structure through the works of European mathematicians.
   a. Create a Fibonacci sequence when given two initial integers.
   b. Investigate Tartaglia's formula for solving cubic equations.

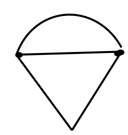
6. Explain the development and applications of logarithms, including contributions of John Napier, Henry Briggs, and the Bernoulli family.

7. Justify the historical significance of the development of multiple perspectives in mathematics.
   Example: Relate the historical development of multiple perspectives to the works of Sir Isaac Newton and Gottfried Wilhelm von Leibniz in the foundations of calculus.

   a. Summarize the significance of René Descartes' Cartesian coordinate system.
   b. Interpret the foundation of analytic geometry with regard to geometric curves and algebraic relationships.

GEOMETRY

8. Solve problems from non-Euclidean geometry, including graph theory, networks, topology, and fractals.
   Examples: Observe the figure to the right to determine if it is traversable, and if it is, describe a path that will traverse it.
   Verify that two objects are topologically equivalent.
   Sketch four iterations of Sierpínski's triangle.



Object-oriented Program Design

9. Analyze works of visual art and architecture for mathematical relationships.
   Examples: Use Leonardo da Vinci's *Vitruvian Man* to explore the golden ratio.
   Identify mathematical patterns in Maurits Cornelis Escher's drawings, including the use of tessellations in art, quilting, paintings, pottery, and architecture.

   a. Summarize the historical development of perspective in art and architecture.

10. Determine the mathematical impact of the ancient Greeks, including Archimedes, Eratosthenes, Euclid, Hypatia, Pythagoras, and the Pythagorean Society.
    Example: Use Euclid's proposition to inscribe a regular hexagon within a circle.

    a. Construct multiple proofs of the Pythagorean Theorem.
    b. Solve problems involving figurate numbers, including triangular and pentagonal numbers.
       Example: Write a sequence of the first 10 triangular numbers and hypothesize a formula for finding the n^{th} triangular number.

11. Describe the development of mathematical tools and their applications.

Computing in Context.

Examples: Use knotted ropes for counting; Napier's bones for multiplication; a slide rule for multiplying and calculating values of trigonometric, exponential, and logarithmic functions; and a graphing calculator for analyzing functions graphically and numerically.

STATISTICS AND PROBABILITY

12. Summarize the history of probability, including the works of Blaise Pascal; Pierre de Fermat; Abraham de Moivre; and Pierre-Simon, marquis de Laplace.

    Example: Discuss the impact of probability on gaming, economics, and insurance.

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PRECALCULUS

Precalculus is a course designed for students who have successfully completed the Algebra II With Trigonometry course. This course is considered to be a prerequisite for success in calculus and college mathematics. Algebraic, graphical, numerical, and verbal analyses are incorporated during investigations of the Precalculus content standards. Parametric equations, polar relations, vector operations, and limits are introduced. Content for this course also includes an expanded study of polynomial and rational functions, conic sections, trigonometric functions, and logarithmic and exponential functions.

Application-based problem solving is an integral part of the course. Instruction should include appropriate use of technology to facilitate continued development of students' higher-order thinking skills.

Students will:

NUMBER AND QUANTITY

The Complex Number System

Represent complex numbers and their operations on the complex plane.

1. (+) Represent complex numbers on the complex plane in rectangular and polar form (including real and imaginary numbers), and explain why the rectangular and polar forms of a given complex number represent the same number. [N-CN4]

2. (+) Represent addition, subtraction, multiplication, and conjugation of complex numbers geometrically on the complex plane; use properties of this representation for computation. [N-CN5]

   Example: $(-1 + \sqrt{3}\,i)^3 = 8$ because $(-1 + \sqrt{3}\,i)$ has modulus 2 and argument 120°.

3. (+) Calculate the distance between numbers in the complex plane as the modulus of the difference, and the midpoint of a segment as the average of the numbers at its endpoints. [N-CN6]

Limits

Understand limits of functions.

4. Determine numerically, algebraically, and graphically the limits of functions at specific values and at infinity.
   a. Apply limits in problems involving convergence and divergence.

Vector and Matrix Quantities

Represent and model with vector quantities.

5. (+) Recognize vector quantities as having both magnitude and direction. Represent vector quantities by directed line segments, and use appropriate symbols for vectors and their magnitudes (e.g., **v**, |**v**|, ||**v**||, v). [N-VM1]

6. (+) Find the components of a vector by subtracting the coordinates of an initial point from the coordinates of a terminal point. [N-VM2]

7. (+) Solve problems involving velocity and other quantities that can be represented by vectors. [N-VM3]

Perform operations on vectors.

8. (+) Add and subtract vectors. [N-VM4]
   a. (+) Add vectors end-to-end, component-wise, and by the parallelogram rule. Understand that the magnitude of a sum of two vectors is typically not the sum of the magnitudes. [N-VM4a]
   b. (+) Given two vectors in magnitude and direction form, determine the magnitude and direction of their sum. [N-VM4b]
   c. (+) Understand vector subtraction **v** – **w** as **v** + (–**w**), where –**w** is the additive inverse of **w**, with the same magnitude as **w** and pointing in the opposite direction. Represent vector subtraction graphically by connecting the tips in the appropriate order, and perform vector subtraction component-wise. [N-VM4c]

9. (+) Multiply a vector by a scalar. [N-VM5]
   a. (+) Represent scalar multiplication graphically by scaling vectors and possibly reversing their direction; perform scalar multiplication component-wise, e.g., as $c(v_x, v_y) = (cv_x, cv_y)$. [N-VM5a]
   b. (+) Compute the magnitude of a scalar multiple c**v** using $||c\mathbf{v}|| = |c|v$. Compute the direction of c**v** knowing that when $|c|v \neq 0$, the direction of c**v** is either along **v** (for $c > 0$) or against **v** (for $c < 0$). [N-VM5b]

Perform operations on matrices and use matrices in applications.

10. (+) Multiply a vector (regarded as a matrix with one column) by a matrix of suitable dimensions to produce another vector. Work with matrices as transformations of vectors. [N-VM11]

11. (+) Work with 2 × 2 matrices as transformations of the plane, and interpret the absolute value of the determinant in terms of area. [N-VM12]

ALGEBRA

Seeing Structure in Expressions

Write expressions in equivalent forms to solve problems.

12. Derive the formula for the sum of a finite geometric series (when the common ratio is not 1), and use the formula to solve problems.* *(Extend to infinite geometric series.)* [A-SSE4]
    Example: Calculate mortgage payments.

Program Implementation. A. Implementation techniques. 1.B Top down development.
Program Analysis. G. Analysis of Algorithms

Arithmetic With Polynomials and Rational Expressions

Use polynomial identities to solve problems.

13. (+) Know and apply the Binomial Theorem for the expansion of $(x + y)^n$ in powers of x and y for a positive integer n, where x and y are any numbers, with coefficients determined, for example, by Pascal's Triangle. (The Binomial Theorem can be proved by mathematical induction or by a combinatorial argument.) [A-APR5]

Reasoning With Equations and Inequalities

Solve systems of equations.

14. (+) Represent a system of linear equations as a single matrix equation in a vector variable. [A-REI8]

FUNCTIONS

Conic Sections

Understand the graphs and equations of conic sections.

15. Create graphs of conic sections, including parabolas, hyperbolas, ellipses, circles, and degenerate conics, from second-degree equations.
    Example: Graph $x^2 – 6x + y^2 – 12y + 41 = 0$ or $y^2 – 4x + 2y + 5 = 0$.

    a. Formulate equations of conic sections from their determining characteristics.
       Example: Write the equation of an ellipse with center (5, -3), a horizontal major axis of length 10, and a minor axis of length 4.

       Answer: $\frac{(x-5)^2}{25} + \frac{(y+3)^2}{4} = 1.$

Interpreting Functions

Interpret functions that arise in applications in terms of the context. ***(Emphasize selection of appropriate models. Understand limits of functions.)***

16. For a function that models a relationship between two quantities, interpret key features of graphs and tables in terms of the quantities, and sketch graphs showing key features given a verbal description of the relationship. (*Key features include intercepts; intervals where the function is increasing, decreasing, positive, or negative; relative maximums and minimums; symmetries; end behavior; and periodicity. Determine odd, even, neither.)** [F-IF4]

17. Calculate and interpret the average rate of change of a function (presented symbolically or as a table) over a specified interval. Estimate the rate of change from a graph.* [F-IF6]

Analyze functions using different representations. ***(Focus on using key features to guide selection of appropriate type of model function with emphasis on piecewise, step, and absolute value. Also emphasize inverse and transformations of polynomials, rational, radical, absolute value, and trigonometric functions.)***

18. Graph functions expressed symbolically, and show key features of the graph, by hand in simple cases and using technology for more complicated cases.* [F-IF7]
    a. Graph square root, cube root, and piecewise-defined functions, including step functions and absolute value functions. [F-IF7b]
    b. Graph polynomial functions, identifying zeros when suitable factorizations are available, and showing end behavior. [F-IF7c]
    c. (+) Graph rational functions, identifying zeros and asymptotes when suitable factorizations are available, and showing end behavior. [F-IF7d]
    d. Graph exponential and logarithmic functions, showing intercepts and end behavior, and trigonometric functions, showing period, midline, and amplitude. [F-IF7e]

Building Functions

Build a function that models a relationship between two quantities.

19. (+) Compose functions. [F-BF1c]
    Example: If $T(y)$ is the temperature in the atmosphere as a function of height, and $h(t)$ is the height of a weather balloon as a function of time, then $T(h(t))$ is the temperature at the location of the weather balloon as a function of time.

Object-Oriented Program Design B. Class Design 1. Design and implement a Class
Object-Oriented Program Design B. Class Design 2. Choose appropriate data representation and algorithms.
Object-Oriented Program Design B. Class Design 3. Apply functional decomposition
Program Implementation. A. Implementation Techniques 1.d Procedural abstraction

Build new functions from existing functions.

20. Determine the inverse of a function and a relation.

21. (+) Verify by composition that one function is the inverse of another. [F-BF4b]

22. (+) Read values of an inverse function from a graph or a table, given that the function has an inverse. [F-BF4c]

23. (+) Produce an invertible function from a non-invertible function by restricting the domain. [F-BF4d]

24. (+) Understand the inverse relationship between exponents and logarithms, and use this relationship to solve problems involving logarithms and exponents. [F-BF5]

Program Analysis. G. Analysis of Algorithms 2. Exact calculation of statement execution counts.

25. Compare effects of parameter changes on graphs of transcendental functions.
    Example: Explain the relationship of the graph $y = e^{x-2}$ to the graph $y = e^x$.

Trigonometric Functions

Recognize attributes of trigonometric functions and solve problems involving trigonometry.

26. Determine the amplitude, period, phase shift, domain, and range of trigonometric functions and their inverses.

27. Use the sum, difference, and half-angle identities to find the exact value of a trigonometric function.

28. Utilize parametric equations by graphing and by converting to rectangular form.
    a. Solve application-based problems involving parametric equations.
    b. Solve applied problems that include sequences with recurrence relations.

Extend the domain of trigonometric functions using the unit circle.

29. (+) Use special triangles to determine geometrically the values of sine, cosine, and tangent for $\frac{\pi}{3}$, $\frac{\pi}{4}$, and $\frac{\pi}{6}$, and use the unit circle to express the values of sine, cosine, and tangent for $\pi - x$, $\pi + x$, and $2\pi - x$ in terms of their values for x, where x is any real number. [F-TF3]

30. (+) Use the unit circle to explain symmetry (odd and even) and periodicity of trigonometric functions. [F-TF4]

Model periodic phenomena with trigonometric functions.

31. (+) Understand that restricting a trigonometric function to a domain on which it is always increasing or always decreasing allows its inverse to be constructed. [F-TF6]

32. (+) Use inverse functions to solve trigonometric equations that arise in modeling contexts; evaluate the solutions using technology, and interpret them in terms of the context.* [F-TF7]

Prove and apply trigonometric identities.

33. Prove the Pythagorean identity $\sin^2(\theta) + \cos^2(\theta) = 1$, and use it to find $\sin(\theta)$, $\cos(\theta)$, or $\tan(\theta)$ given $\sin(\theta)$, $\cos(\theta)$, or $\tan(\theta)$ and the quadrant of the angle. [F-TF8]

34. (+) Prove the addition and subtraction formulas for sine, cosine, and tangent, and use them to solve problems. [F-TF9]

GEOMETRY

Similarity, Right Triangles, and Trigonometry

Apply trigonometry to general triangles.

35. (+) Derive the formula $A = (\frac{1}{2})ab \sin(C)$ for the area of a triangle by drawing an auxiliary line from a vertex perpendicular to the opposite side. (*Apply formulas previously derived in Geometry.*) [G-SRT9]

Expressing Geometric Properties With Equations

Translate between the geometric description and the equation for a conic section.

36. (+) Derive the equations of a parabola given a focus and directrix. [G-GPE2]

37. (+) Derive the equations of ellipses and hyperbolas given the foci, using the fact that the sum or difference of distances from the foci is constant. [G-GPE3]

Explain volume formulas and use them to solve problems.

38. (+) Give an informal argument using Cavalieri's principle for the formulas for the volume of a sphere and other solid figures. [G-GMD2]

STATISTICS AND PROBABILITY

Interpreting Categorical and Quantitative Data

Summarize, represent, and interpret data on a single count or measurement variable.

39. Use statistics appropriate to the shape of the data distribution to compare center (median, mean) and spread (interquartile range, standard deviation) of two or more different data sets. *(Focus on increasing rigor using standard deviation.)* [S-ID2]

40. Interpret differences in shape, center, and spread in the context of the data sets, accounting for possible effects of extreme data points (outliers). *(Identify uniform, skewed, and normal distributions in a set of data. Determine the quartiles and interquartile range for a set of data.)* [S-ID3]

41. Use the mean and standard deviation of a data set to fit it to a normal distribution and to estimate population percentages. Recognize that there are data sets for which such a procedure is not appropriate. Use calculators, spreadsheets, and tables to estimate areas under the normal curve. [S-ID4]

Interpret linear models.

42. Compute (using technology) and interpret the correlation coefficient of a linear fit. [S-ID8]

43. Distinguish between correlation and causation. [S-ID9]

Making Inferences and Justifying Conclusions

Understand and evaluate random processes underlying statistical experiments.

44. Understand statistics as a process for making inferences about population parameters based on a random sample from that population. [S-IC1]

45. Decide if a specified model is consistent with results from a given data-generating process, e.g., using simulation. [S-IC2]
    Example: A model says a spinning coin falls heads up with probability 0.5. Would a result of 5 tails in a row cause you to question the model?

Object-Oriented Program Design B. Class Design
Object-Oriented Program Design B. Class Design 1. Design and implement a Class
Object-Oriented Program Design B. Class Design 4. Extend a given class using inheritance
Program Implementation. Implementation Techniques 1.A Object-oriented development
Program Analysis D. Extend existing code using inheritance.
Program Analysis F. Reason about programs.
Standard Data Structures D. Arrays

Make inferences and justify conclusions from sample surveys, experiments, and observational studies.

46. Recognize the purposes of and differences among sample surveys, experiments, and observational studies; explain how randomization relates to each. [S-IC3]

47. Use data from a sample survey to estimate a population mean or proportion; develop a margin of error through the use of simulation models for random sampling. [S-IC4]

48. Use data from a randomized experiment to compare two treatments; use simulations to decide if differences between parameters are significant. [S-IC5]

49. Evaluate reports based on data. [S-IC6]

Using Probability to Make Decisions

Calculate expected values and use them to solve problems.

50. (+) Define a random variable for a quantity of interest by assigning a numerical value to each event in a sample space; graph the corresponding probability distribution using the same graphical displays as for data distributions. [S-MD1]

Program Implementation. Java Library classes

51. (+) Calculate the expected value of a random variable; interpret it as the mean of the probability distribution. [S-MD2]

Program Implementation. Java Library classes

52. (+) Develop a probability distribution for a random variable defined for a sample space in which theoretical probabilities can be calculated; find the expected value. [S-MD3]
    Example: Find the theoretical probability distribution for the number of correct answers obtained by guessing on all five questions of a multiple-choice test where each question has four choices, and find the expected grade under various grading schemes.

53. (+) Develop a probability distribution for a random variable defined for a sample space in which probabilities are assigned empirically; find the expected value. [S-MD4]
    Example: Find a current data distribution on the number of television sets per household in the United States, and calculate the expected number of sets per household. How many television sets would you expect to find in 100 randomly selected households?

Use probability to evaluate outcomes of decisions.

54. (+) Weigh the possible outcomes of a decision by assigning probabilities to payoff values and finding expected values. [S-MD5]
    a. Find the expected payoff for a game of chance. [S-MD5a]
        Examples: Find the expected winnings from a state lottery ticket or a game at a fast-food restaurant.
    b. Evaluate and compare strategies on the basis of expected values. [S-MD5b]
        Example: Compare a high-deductible versus a low-deductible automobile insurance policy using various, but reasonable, chances of having a minor or a major accident.

Object-Oriented Program Design B. Class Design
Program Implementation. Java Library classes

ANALYTICAL MATHEMATICS

Analytical Mathematics is a course designed for students who have successfully completed the Algebra II With Trigonometry course. It is considered to be parallel in rigor to Precalculus. This course provides a structured introduction to important areas of emphasis in most postsecondary studies that pursue a concentration in mathematics. Linear algebra, logic, vectors, and matrices are topics that are given more in-depth coverage than in previous courses. Application-based problem solving is an integral part of this course. To assist students with numerical and graphical analysis, the use of advanced technological tools is highly recommended.

While this course may be taken either prior to or after Precalculus, it is recommended that students who are interested in postsecondary studies in engineering successfully complete the Precalculus course as well as, where available, an Advanced Placement or International Baccalaureate calculus course.

Students will:

NUMBER AND QUANTITY

Vector and Matrix Quantities

Represent and model with vector quantities.

1. (+) Recognize vector quantities as having both magnitude and direction. Represent vector quantities by directed line segments, and use appropriate symbols for vectors and their magnitudes (e.g., **v**, |**v**|, ||**v**||), including the use of eigen-values and eigen-vectors. [N-VM1]

2. (+) Solve problems involving velocity and other quantities that can be represented by vectors, including navigation (e.g., airplane, aerospace, oceanic). [N-VM3]

3. (+) Add vectors end-to-end, component-wise, and by the parallelogram rule. Understand that the magnitude of a sum of two vectors is typically not the sum of the magnitudes. Find the dot product and the cross product of vectors. [N-VM4a]

4. (+) Given two vectors in magnitude and direction form, determine the magnitude and direction of their sum, including vectors in complex vector spaces. [N-VM4b]

5. (+) Understand vector subtraction **v** – **w** as **v** + (–**w**), where (–**w**) is the additive inverse of **w**, with the same magnitude as **w** and pointing in the opposite direction. Represent vector subtraction graphically by connecting the tips in the appropriate order, and perform vector subtraction component-wise, including vectors in complex vector spaces. [N-VM4c]

Perform operations on matrices and use matrices in applications.

6. (+) Use matrices to represent and manipulate data, e.g., to represent payoffs or incidence relationships in a network, including linear programming. [N-VM6]

Standard Data Structures. A. Simple data types D. Arrays.

7. (+) Multiply matrices by scalars to produce new matrices, e.g., as when all of the payoffs in a game are doubled, including rotation matrices. [N-VM7]

Standard Data Structures. A. Simple data types D. Arrays

8. (+) Understand that the zero and identity matrices play a role in matrix addition and multiplication similar to the role of 0 and 1 in the real numbers. The determinant of a square matrix is nonzero if and only if the matrix has a multiplicative inverse. Solve matrix equations using augmented matrices. [N-VM10]

9. (+) Multiply a vector (regarded as a matrix with one column) by a matrix of suitable dimensions to produce another vector. Work with matrices as transformations of vectors, including matrices larger than 2 × 2. [N-VM11]

10. (+) Work with 2 × 2 matrices as transformations of the plane, and interpret the absolute value of the determinant in terms of area. Solve matrix application problems using reduced row echelon form. [N-VM12]

Complex Numbers

Use complex numbers in polynomial identities and equations.

11. (+) Know the Fundamental Theorem of Algebra; show that it is true for quadratic polynomials. Understand the importance of using complex numbers in graphing functions on the Cartesian or complex plane. [N-CN9]

Limits

Understand limits of functions.

12. Calculate the limit of a sequence, of a function, and of an infinite series.

ALGEBRA

Seeing Structure in Expressions

13. Use the laws of Boolean Algebra to describe true/false circuits. Simplify Boolean expressions using the relationships between conjunction, disjunction, and negation operations.

Program Implementation. B. Programming constructs. 4. Control

14. Use logic symbols to write truth tables.

Program Implementation. B. Programming constructs. 4. Control

Arithmetic With Polynomials and Rational Functions

15. Reduce the degree of either the numerator or denominator of a rational function by using partial fraction decomposition or partial fraction expansion.

FUNCTIONS

Trigonometric Functions

Extend the domain of trigonometric functions using the unit circle.

16. (+) Use the unit circle to explain symmetry (odd and even) and periodicity of trigonometric functions. [F-TF4].

Apply trigonometry to general triangles.

17. (+) Prove the Law of Sines and the Law of Cosines and use them to solve problems. Understand Law of Sines = 2*r*, where *r* is the radius of the circumscribed circle of the triangle. Apply the Law of Tangents. [G-SRT10]

18. Apply Euler's and deMoivre's formulas as links between complex numbers and trigonometry.

	Computer Science Principles	Algebra II	Algebra II with Trig	Discrete Mathematics	Mathematical Investigations	Precalculus	Analytical Mathematics
Learning Objectives	Description						
	Big Idea I: Creativity. The student can…						
LO 1.1.1	use computing tools and techniques to create artifacts				2		
LO 1.1.2	collaborate in the creation of the computational artifact						
LO 1.1.3	analyze computational artifacts			1	2		
LO 1.2.1	use computing tools and techniques for creative expression						
LO 1.3.1	use programming as a creative tool						
	Big Idea II: Abstraction. The student can…						
LO 2.1.1	describe the combination of abstractions used to represent data						
LO 2.1.2	explain how binary sequences are used to represent digital data	4	4				
LO 2.2.1	develop an abstraction						
LO 2.2.2	use multiple levels of abstraction in computation						
LO 2.3.1	use models and simulations to raise and answer questions	37, 38	41, 42			39,40, 41, 42, 43	
	Big Idea III: Data. The student can…						

LO 3.1.1	use computers to process information to gain insight and knowledge			13	11	39, 40, 41,42, 43, 46, 53	
LO 3.1.2	collaborate when processing information to gain insight and knowledge			12, 13		39, 40, 41,42, 43, 44	
LO 3.1.3	communicate insight and knowledge gained from using computer programs to process information			13		39, 40, 41,42, 43	
LO 3.2.1	use computing to facilitate exploration and the discovery of connections in information			10		39, 40, 41,42, 43	
LO 3.2.2	use large datasets to explore and discover information and knowledge	42	46	10, 13		39, 40, 41,42, 43, 47, 48	
LO 3.3.1	analyze the considerations involved in the computational manipulation of information					39, 40, 41,42, 43, 45,49, 54	
	Big Idea IV Algorithms. The student can…						
LO 4.1.1	develop an algorithm designed to be implemented to run on a computer			7a, 8			
LO 4.2.1	express an algorithm in a language.			1	8		
LO 4.3.1	appropriately connect problems and potential algorithmic solutions.			7a, 8			
LO 4.4.1	evaluate algorithms analytically and empirically			1	6		
	Big Idea V Programming. The student can…						
LO 5.1.1	explain how programs implement algorithms			8			
LO 5.2.1	use abstraction to manage complexity in programs			9			
LO 5.3.1	evaluate a program for correctness			2,3	5a		

LO 5.3.2	develop a correct program.			12			
LO 5.3.3	collaborate to solve a problem using programming						
LO 5.4.1	employ appropriate mathematical and logical concepts in programming	39	43	2,3, 11			13,14
	Big Idea VI Internet. The student can…						
LO 6.1.1	explain the abstractions in the Internet and how the Internet functions.				11		
LO 6.2.1	explain characteristics of the Internet and the systems built on it.				8		
LO 6.2.2	analyze how characteristics of the Internet and systems built on it influence their use.						
LO 6.3.1	connect the concern of cybersecurity with the Internet and systems built on it			1			
	Big Idea VII Impact. The student can…						
LO 7.1.1	analyze how computing affects communication, interaction, and cognition						
LO 7.1.2	collaborate as a part of a process that scales						
LO 7.2.1	connect computing with innovations in other fields				2, 5a, 9		
LO 7.3.1	analyze the beneficial and harmful effects of computing			1			
LO 7.4.1	connect computing within economic, social, and cultural contexts	14	14		12	12	

ALGEBRA II

Algebra II is a terminating course designed to extend students' algebraic knowledge and skills beyond Algebra I. Students are encouraged to solve problems using a variety of methods that promote the development of improved communication skills and foster a deeper understanding of mathematics. To help students appreciate the power of algebra, application-based problems are incorporated throughout the course. The use of appropriate technology is also encouraged for numerical and graphical investigations.

In contrast to the Algebra II With Trigonometry course, Algebra II does not meet the graduation requirements for the Alabama High School Diploma with Advanced Academic Endorsement due to the fact that it does not contain trigonometry content. Algebra II With Trigonometry or Algebra II is required to complete the graduation requirements for the Alabama High School Diploma. This course does not provide sufficient background to prepare students to pursue higher-level mathematics courses. The prerequisites for Algebra II are Algebra I and Geometry.

Students will:

NUMBER AND QUANTITY

The Complex Number System

Perform arithmetic operations with complex numbers.

1. Know there is a complex number i such that $i^2 = -1$, and every complex number has the form $a + bi$ with a and b real. [N-CN1]

2. Use the relation $i^2 = -1$ and the commutative, associative, and distributive properties to add, subtract, and multiply complex numbers. [N-CN2]

3. (+) Find the conjugate of a complex number; use conjugates to find moduli and quotients of complex numbers. [N-CN3]

Use complex numbers in polynomial identities and equations. ***(Polynomials with real coefficients.)***

4. Solve quadratic equations with real coefficients that have complex solutions. [N-CN7]

LO 2.1.2	explain how binary sequences are used to represent digital data

5. (+) Extend polynomial identities to the complex numbers. [N-CN8]
   Example: Rewrite $x^2 + 4$ as $(x + 2i)(x - 2i)$.

6. (+) Know the Fundamental Theorem of Algebra; show that it is true for quadratic polynomials. [N-CN9]

Vector and Matrix Quantities

Perform operations on matrices and use matrices in applications.

7. (+) Use matrices to represent and manipulate data, e.g., to represent payoffs or incidence relationships in a network. (*Use technology to approximate roots.)* [N-VM6]

8. (+) Multiply matrices by scalars to produce new matrices, e.g., as when all of the payoffs in a game are doubled. [N-VM7]

9. (+) Add, subtract, and multiply matrices of appropriate dimensions. [N-VM8]

10. (+) Understand that, unlike multiplication of numbers, matrix multiplication for square matrices is not a commutative operation, but still satisfies the associative and distributive properties. [N-VM9]

11. (+) Understand that the zero and identity matrices play a role in matrix addition and multiplication similar to the role of 0 and 1 in the real numbers. The determinant of a square matrix is nonzero if and only if the matrix has a multiplicative inverse. [N-VM10]

ALGEBRA

Seeing Structure in Expressions

Interpret the structure of expressions. (*Polynomial and rational.)*

12. Interpret expressions that represent a quantity in terms of its context.* [A-SSE1]
    a. Interpret parts of an expression such as terms, factors, and coefficients. [A-SSE1a]
    b. Interpret complicated expressions by viewing one or more of their parts as a single entity. [A-SSE1b]
        Example: Interpret $P(1+r)^n$ as the product of P and a factor not depending on P.

13. Use the structure of an expression to identify ways to rewrite it. [A-SSE2]
    Example: See $x^4 - y^4$ as $(x^2)^2 - (y^2)^2$, thus recognizing it as a difference of squares that can be factored as $(x^2 - y^2)(x^2 + y^2)$.

Write expressions in equivalent forms to solve problems.

14. Derive the formula for the sum of a finite geometric series (when the common ratio is not 1), and use the formula to solve problems.* [A-SSE4]
    Example: Calculate mortgage payments.

LO 7.4.1	connect computing within economic, social, and cultural contexts

Arithmetic With Polynomials and Rational Expressions

Perform arithmetic operations on polynomials. *(Beyond quadratic.)*

15. Understand that polynomials form a system analogous to the integers; namely, they are closed under the operations of addition, subtraction, and multiplication; add, subtract, and multiply polynomials. [A-APR1]

Understand the relationship between zeros and factors of polynomials.

16. Know and apply the Remainder Theorem: For a polynomial $p(x)$ and a number a, the remainder on division by $x - a$ is $p(a)$, so $p(a) = 0$ if and only if $(x - a)$ is a factor of $p(x)$. [A-APR2]

17. Identify zeros of polynomials when suitable factorizations are available, and use the zeros to construct a rough graph of the function defined by the polynomial. [A-APR3]

Use polynomial identities to solve problems.

18. Prove polynomial identities and use them to describe numerical relationships. [A-APR4]
    Example: The polynomial identity $(x^2 + y^2)^2 = (x^2 - y^2)^2 + (2xy)^2$ can be used to generate Pythagorean triples.

Rewrite rational expressions. ***(Linear and quadratic denominators.)***

19. Rewrite simple rational expressions in different forms; write $a(x)/b(x)$ in the form $q(x) + r(x)/b(x)$, where $a(x)$, $b(x)$, $q(x)$, and $r(x)$ are polynomials with the degree of $r(x)$ less than the degree of $b(x)$, using inspection, long division, or for the more complicated examples, a computer algebra system. [A-APR6]

Creating Equations*

Create equations that describe numbers or relationships. ***(Equations using all available types of expressions, including simple root functions.)***

20. Create equations and inequalities in one variable and use them to solve problems. *Include equations arising from linear and quadratic functions, and simple rational and exponential functions.* [A-CED1]

21. Create equations in two or more variables to represent relationships between quantities; graph equations on coordinate axes with labels and scales. [A-CED2]

22. Represent constraints by equations or inequalities, and by systems of equations and/or inequalities, and interpret solutions as viable or nonviable options in a modeling context. [A-CED3]
    Example: Represent inequalities describing nutritional and cost constraints on combinations of different foods.

23. Rearrange formulas to highlight a quantity of interest, using the same reasoning as in solving equations. [A-CED4]
    Example: Rearrange Ohm's law $V = IR$ to highlight resistance R.

Reasoning With Equations and Inequalities

Understand solving equations as a process of reasoning and explain the reasoning. ***(Simple rational and radical.)***

24. Solve simple rational and radical equations in one variable, and give examples showing how extraneous solutions may arise. [A-REI2]

Solve equations and inequalities in one variable.

25. Recognize when the quadratic formula gives complex solutions, and write them as $a \pm bi$ for real numbers a and b. [A-REI4b]

Solve systems of equations.

26. (+) Find the inverse of a matrix if it exists and use it to solve systems of linear equations (using technology for matrices of dimension 3 × 3 or greater). [A-REI9]

Represent and solve equations and inequalities graphically. ***(Combine polynomial, rational, radical, absolute value, and exponential functions.)***

27. Explain why the x-coordinates of the points where the graphs of the equations $y = f(x)$ and $y = g(x)$ intersect are the solutions of the equation $f(x) = g(x)$; find the solutions approximately, e.g., using technology to graph the functions, make tables of values, or find successive approximations. Include cases where $f(x)$ and/or $g(x)$ are linear, polynomial, rational, absolute value, exponential, and logarithmic functions.* [A-REI11]

Conic Sections

Understand the graphs and equations of conic sections. ***(Emphasize understanding graphs and equations of circles and parabolas.)***

28. Create graphs of conic sections, including parabolas, hyperbolas, ellipses, circles, and degenerate conics, from second-degree equations.
    Example: Graph $x^2 - 6x + y^2 - 12y + 41 = 0$ or $y^2 - 4x + 2y + 5 = 0$.

    a. Formulate equations of conic sections from their determining characteristics.
       Example: Write the equation of an ellipse with center (5, -3), a horizontal major axis of length 10, and a minor axis of length 4.
       Answer: $\frac{(x-5)^2}{25} + \frac{(y+3)^2}{4} = 1.$

FUNCTIONS

Interpreting Functions

Interpret functions that arise in applications in terms of the context. ***(Emphasize selection of appropriate models.)***

29. Relate the domain of a function to its graph and, where applicable, to the quantitative relationship it describes.* [F-IF5]
    Example: If the function $h(n)$ gives the number of person-hours it takes to assemble n engines in a factory, then the positive integers would be an appropriate domain for the function.

Analyze functions using different representations. ***(Focus on using key features to guide selection of appropriate type of model function.)***

30. Graph functions expressed symbolically, and show key features of the graph, by hand in simple cases and using technology for more complicated cases.* [F-IF7]
    a. Graph square root, cube root, and piecewise-defined functions, including step functions and absolute value functions. [F-IF7b]

b. Graph polynomial functions, identifying zeros when suitable factorizations are available, and showing end behavior. [F-IF7c]
c. Graph exponential and logarithmic functions, showing intercepts and end behavior, and trigonometric functions, showing period, midline, and amplitude. [F-IF7e]

31. Write a function defined by an expression in different but equivalent forms to reveal and explain different properties of the function. [F-IF8]

32. Compare properties of two functions each represented in a different way (algebraically, graphically, numerically in tables, or by verbal descriptions). [F-IF9]
Example: Given a graph of one quadratic function and an algebraic expression for another, say which has the larger maximum.

Building Functions

Build a function that models a relationship between two quantities. ***(Include all types of functions studied.)***

33. Write a function that describes a relationship between two quantities.* [F-BF1]
a. Combine standard function types using arithmetic operations. [F-BF1b]
Example: Build a function that models the temperature of a cooling body by adding a constant function to a decaying exponential, and relate these functions to the model.

Build new functions from existing functions. ***(Include simple radical, rational, and exponential functions; emphasize common effect of each transformation across function types.)***

34. Identify the effect on the graph of replacing $f(x)$ by $f(x) + k$, $k\,f(x)$, $f(kx)$, and $f(x + k)$ for specific values of k (both positive and negative); find the value of k given the graphs. Experiment with cases and illustrate an explanation of the effects on the graph using technology. Include recognizing even and odd functions from their graphs and algebraic expressions for them. [F-BF3]

35. Find inverse functions. [F-BF4]
a. Solve an equation of the form $f(x) = c$ for a simple function f that has an inverse, and write an expression for the inverse. [F-BF4a]
Example: $f(x) = 2x^3$ or $f(x) = (x+1)/(x-1)$ for $x \neq 1$.

Linear, Quadratic, and Exponential Models*

Construct and compare linear, quadratic, and exponential models and solve problems. ***(Logarithms as solutions for exponentials.)***

36. For exponential models, express as a logarithm the solution to $ab^{ct} = d$ where a, c, and d are numbers, and the base b is 2, 10, or e; evaluate the logarithm using technology. [F-LE4]

STATISTICS AND PROBABILITY

Using Probability to Make Decisions

Use probability to evaluate outcomes of decisions. ***(Include more complex situations.)***

37. (+) Use probabilities to make fair decisions (e.g., drawing by lots, using a random number generator). [S-MD6]

LO 2.3.1	use models and simulations to raise and answer questions

38. (+) Analyze decisions and strategies using probability concepts (e.g., product testing, medical testing, pulling a hockey goalie at the end of a game). [S-MD7]

LO 2.3.1	use models and simulations to raise and answer questions

Conditional Probability and the Rules of Probability

Understand independence and conditional probability and use them to interpret data.
(Link to data from simulations or experiments.)

39. Describe events as subsets of a sample space (the set of outcomes), using characteristics (or categories) of the outcomes, or as unions, intersections, or complements of other events (“or,” “and,” “not”). [S-CP1]

LO 5.4.1	employ appropriate mathematical and logical concepts in programming

40. Understand the conditional probability of A given B as $P(A \text{ and } B)/P(B)$, and interpret independence of A and B as saying that the conditional probability of A given B is the same as the probability of A, and the conditional probability of B given A is the same as the probability of B. [S-CP3]

41. Construct and interpret two-way frequency tables of data when two categories are associated with each object being classified. Use the two-way table as a sample space to decide if events are independent and to approximate conditional probabilities. [S-CP4]

    Example: Collect data from a random sample of students in your school on their favorite subject among mathematics, science, and English. Estimate the probability that a randomly selected student from your school will favor science given that the student is in tenth grade. Do the same for other subjects and compare the results.

42. Recognize and explain the concepts of conditional probability and independence in everyday language and everyday situations. [S-CP5]

    Example: Compare the chance of having lung cancer if you are a smoker with the chance of being a smoker if you have lung cancer.

Use the rules of probability to compute probabilities of compound events in a uniform probability model.

LO 3.2.2	use large datasets to explore and discover information and knowledge

43. Find the conditional probability of A given B as the fraction of B's outcomes that also belong to A, and interpret the answer in terms of the model. [S-CP6]

44. Apply the Addition Rule, $P(A \text{ or } B) = P(A) + P(B) - P(A \text{ and } B)$, and interpret the answer in terms of the model. [S-CP7]

45. (+) Apply the general Multiplication Rule in a uniform probability model, $P(A \text{ and } B) = P(A)P(B|A) = P(B)P(A|B)$, and interpret the answer in terms of the model. [S-CP8]

46. (+) Use permutations and combinations to compute probabilities of compound events and solve problems. [S-CP9]

ALGEBRA II WITH TRIGONOMETRY

Algebra II With Trigonometry is a course designed to extend students' knowledge of Algebra I with additional algebraic and trigonometric content. Mastery of the content standards for this course is necessary for student success in higher-level mathematics. The use of appropriate technology is encouraged for numerical and graphical investigations that enhance analytical comprehension.

Algebra II With Trigonometry is required for all students pursuing the Alabama High School Diploma with Advanced Academic Endorsement. Prerequisites for this course are Algebra I and Geometry. If a student chooses to take the Algebraic Connections course, it must be taken prior to the Algebra II With Trigonometry course.

Students will:

NUMBER AND QUANTITY

The Complex Number System

Perform arithmetic operations with complex numbers.

1. Know there is a complex number i such that $i^2 = -1$, and every complex number has the form $a + bi$ with a and b real. [N-CN1]

2. Use the relation $i^2 = -1$ and the commutative, associative, and distributive properties to add, subtract, and multiply complex numbers. [N-CN2]

3. (+) Find the conjugate of a complex number; use conjugates to find moduli and quotients of complex numbers. [N-CN3]

Use complex numbers in polynomial identities and equations. ***(Polynomials with real coefficients.)***

4. Solve quadratic equations with real coefficients that have complex solutions. [N-CN7]

LO 2.1.2	explain how binary sequences are used to represent digital data

5. (+) Extend polynomial identities to the complex numbers.
   Example: Rewrite $x^2 + 4$ as $(x + 2i)(x - 2i)$. [N-CN8]

6. (+) Know the Fundamental Theorem of Algebra; show that it is true for quadratic polynomials. [N-CN9]

Vector and Matrix Quantities

Perform operations on matrices and use matrices in applications.

7. (+) Use matrices to represent and manipulate data, e.g., to represent payoffs or incidence relationships in a network. (*Use technology to approximate roots.)* [N-VM6]

8. (+) Multiply matrices by scalars to produce new matrices, e.g., as when all of the payoffs in a game are doubled. [N-VM7]

9. (+) Add, subtract, and multiply matrices of appropriate dimensions. [N-VM8]

10. (+) Understand that, unlike multiplication of numbers, matrix multiplication for square matrices is not a commutative operation, but still satisfies the associative and distributive properties. [N-VM9]

11. (+) Understand that the zero and identity matrices play a role in matrix addition and multiplication similar to the role of 0 and 1 in the real numbers. The determinant of a square matrix is nonzero if and only if the matrix has a multiplicative inverse. [N-VM10]

LO 7.4.1	connect computing within economic, social, and cultural contexts

ALGEBRA

Seeing Structure in Expressions

Interpret the structure of expressions. (*Polynomial and rational.)*

12. Interpret expressions that represent a quantity in terms of its context.* [A-SSE1]
    a. Interpret parts of an expression such as terms, factors, and coefficients. [A-SSE1a]
    b. Interpret complicated expressions by viewing one or more of their parts as a single entity. [A-SSE1b]
        Example: Interpret $P(1+r)^n$ as the product of P and a factor not depending on P.

13. Use the structure of an expression to identify ways to rewrite it. [A-SSE2]
    Example: See $x^4 - y^4$ as $(x^2)^2 - (y^2)^2$, thus recognizing it as a difference of squares that can be factored as $(x^2 - y^2)(x^2 + y^2)$.

Write expressions in equivalent forms to solve problems.

14. Derive the formula for the sum of a finite geometric series (when the common ratio is not 1), and use the formula to solve problems.* [A-SSE4]
    Example: Calculate mortgage payments.

LO 7.4.1	connect computing within economic, social, and cultural contexts

Arithmetic With Polynomials and Rational Expressions

Perform arithmetic operations on polynomials. *(Beyond quadratic.)*

15. Understand that polynomials form a system analogous to the integers; namely, they are closed under the operations of addition, subtraction, and multiplication; add, subtract, and multiply polynomials. [A-APR1]

Understand the relationship between zeros and factors of polynomials.

16. Know and apply the Remainder Theorem: For a polynomial $p(x)$ and a number a, the remainder on division by $x - a$ is $p(a)$, so $p(a) = 0$ if and only if $(x - a)$ is a factor of $p(x)$. [A-APR2]

17. Identify zeros of polynomials when suitable factorizations are available, and use the zeros to construct a rough graph of the function defined by the polynomial. [A-APR3]

Use polynomial identities to solve problems.

18. Prove polynomial identities and use them to describe numerical relationships. [A-APR4]
    Example: The polynomial identity $(x^2 + y^2)^2 = (x^2 - y^2)^2 + (2xy)^2$ can be used to generate Pythagorean triples.

Rewrite rational expressions. ***(Linear and quadratic denominators.)***

19. Rewrite simple rational expressions in different forms; write $a(x)/b(x)$ in the form $q(x) + r(x)/b(x)$, where $a(x)$, $b(x)$, $q(x)$, and $r(x)$ are polynomials with the degree of $r(x)$ less than the degree of $b(x)$, using inspection, long division, or for the more complicated examples, a computer algebra system. [A-APR6]

Creating Equations*

Create equations that describe numbers or relationships. ***(Equations using all available types of expressions, including simple root functions.)***

20. Create equations and inequalities in one variable and use them to solve problems. *Include equations arising from linear and quadratic functions, and simple rational and exponential functions*. [A-CED1]

21. Create equations in two or more variables to represent relationships between quantities; graph equations on coordinate axes with labels and scales. [A-CED2]

22. Represent constraints by equations or inequalities, and by systems of equations and/or inequalities, and interpret solutions as viable or nonviable options in a modeling context. [A-CED3]
    Example: Represent inequalities describing nutritional and cost constraints on combinations of different foods.

23. Rearrange formulas to highlight a quantity of interest, using the same reasoning as in solving equations. [A-CED4]
    Example: Rearrange Ohm's law $V = IR$ to highlight resistance R.

Reasoning With Equations and Inequalities

Understand solving equations as a process of reasoning, and explain the reasoning. ***(Simple rational and radical.)***

24. Solve simple rational and radical equations in one variable, and give examples showing how extraneous solutions may arise. [A-REI2]

Solve equations and inequalities in one variable.

25. Recognize when the quadratic formula gives complex solutions, and write them as $a \pm bi$ for real numbers a and b. [A-REI4b]

Solve systems of equations.

26. (+) Find the inverse of a matrix if it exists and use it to solve systems of linear equations (using technology for matrices of dimension 3 × 3 or greater). [A-REI9]

Represent and solve equations and inequalities graphically. ***(Combine polynomial, rational, radical, absolute value, and exponential functions.)***

27. Explain why the x-coordinates of the points where the graphs of the equations $y = f(x)$ and $y = g(x)$ intersect are the solutions of the equation $f(x) = g(x)$; find the solutions approximately, e.g., using technology to graph the functions, make tables of values, or find successive approximations. Include cases where $f(x)$ and/or $g(x)$ are linear, polynomial, rational, absolute value, exponential, and logarithmic functions.* [A-REI11]

Conic Sections

Understand the graphs and equations of conic sections. ***(Emphasize understanding graphs and equations of circles and parabolas.)***

28. Create graphs of conic sections, including parabolas, hyperbolas, ellipses, circles, and degenerate conics, from second-degree equations.
    Example: Graph $x^2 - 6x + y^2 - 12y + 41 = 0$ or $y^2 - 4x + 2y + 5 = 0$.

    a. Formulate equations of conic sections from their determining characteristics.
       Example: Write the equation of an ellipse with center (5, -3), a horizontal major axis of length 10, and a minor axis of length 4.
       Answer: $\frac{(x-5)^2}{25} + \frac{(y+3)^2}{4} = 1.$

FUNCTIONS

Interpreting Functions

Interpret functions that arise in applications in terms of the context. ***(Emphasize selection of appropriate models.)***

29. Relate the domain of a function to its graph and, where applicable, to the quantitative relationship it describes.* [F-IF5]
    Example: If the function $h(n)$ gives the number of person-hours it takes to assemble n engines in a factory, then the positive integers would be an appropriate domain for the function.

Analyze functions using different representations. ***(Focus on using key features to guide selection of appropriate type of model function.)***

30. Graph functions expressed symbolically, and show key features of the graph, by hand in simple cases and using technology for more complicated cases.* [F-IF7]
    a. Graph square root, cube root, and piecewise-defined functions, including step functions and absolute value functions. [F-IF7b]
    b. Graph polynomial functions, identifying zeros when suitable factorizations are available, and showing end behavior. [F-IF7c]
    c. Graph exponential and logarithmic functions, showing intercepts and end behavior, and trigonometric functions, showing period, midline, and amplitude. [F-IF7e]

31. Write a function defined by an expression in different but equivalent forms to reveal and explain different properties of the function. [F-IF8]

32. Compare properties of two functions each represented in a different way (algebraically, graphically, numerically in tables, or by verbal descriptions). [F-IF9]
    Example: Given a graph of one quadratic function and an algebraic expression for another, say which has the larger maximum.

Building Functions

Build a function that models a relationship between two quantities. ***(Include all types of functions studied.)***

33. Write a function that describes a relationship between two quantities.* [F-BF1]
    a. Combine standard function types using arithmetic operations. [F-BF1b]
        Example: Build a function that models the temperature of a cooling body by adding a constant function to a decaying exponential, and relate these functions to the model.

Build new functions from existing functions. ***(Include simple radical, rational, and exponential functions; emphasize common effect of each transformation across function types.)***

34. Identify the effect on the graph of replacing $f(x)$ by $f(x) + k$, $k\,f(x)$, $f(kx)$, and $f(x + k)$ for specific values of k (both positive and negative); find the value of k given the graphs. Experiment with cases and illustrate an explanation of the effects on the graph using technology. Include recognizing even and odd functions from their graphs and algebraic expressions for them. [F-BF3]

35. Find inverse functions. [F-BF4]
    a. Solve an equation of the form $f(x) = c$ for a simple function f that has an inverse, and write an expression for the inverse. [F-BF4a]
        Example: $f(x) = 2x^3$ or $f(x) = (x+1)/(x-1)$ for $x \neq 1$.

Linear, Quadratic, and Exponential Models*

Construct and compare linear, quadratic, and exponential models and solve problems. ***(Logarithms as solutions for exponentials.)***

36. For exponential models, express as a logarithm the solution to $ab^{ct} = d$ where a, c, and d are numbers, and the base b is 2, 10, or e; evaluate the logarithm using technology. [F-LE4]

Trigonometric Functions

Extend the domain of trigonometric functions using the unit circle.

37. Understand radian measure of an angle as the length of the arc on the unit circle subtended by the angle. [F-TF1]

38. Explain how the unit circle in the coordinate plane enables the extension of trigonometric functions to all real numbers, interpreted as radian measures of angles traversed counterclockwise around the unit circle. [F-TF2]

39. Define the six trigonometric functions using ratios of the sides of a right triangle, coordinates on the unit circle, and the reciprocal of other functions.

Model periodic phenomena with trigonometric functions.

40. Choose trigonometric functions to model periodic phenomena with specified amplitude, frequency, and midline.* [F-TF5]

STATISTICS AND PROBABILITY

Using Probability to Make Decisions

Use probability to evaluate outcomes of decisions. (*Include more complex situations.*)

41. (+) Use probabilities to make fair decisions (e.g., drawing by lots, using a random number generator). [S-MD6]

LO 2.3.1	use models and simulations to raise and answer questions

42. (+) Analyze decisions and strategies using probability concepts (e.g., product testing, medical testing, pulling a hockey goalie at the end of a game). [S-MD7]

LO 2.3.1	use models and simulations to raise and answer questions

Conditional Probability and the Rules of Probability

Understand independence and conditional probability and use them to interpret data. ***(Link to data from simulations or experiments.)***

43. Describe events as subsets of a sample space (the set of outcomes), using characteristics (or categories) of the outcomes, or as unions, intersections, or complements of other events ("or," "and," "not"). [S-CP1]

LO 5.4.1	employ appropriate mathematical and logical concepts in programming

44. Understand the conditional probability of *A* given *B* as *P*(*A* and *B*)/*P*(*B*), and interpret independence of *A* and *B* as saying that the conditional probability of *A* given *B* is the same as the probability of *A*, and the conditional probability of *B* given *A* is the same as the probability of *B*. [S-CP3]

45. Construct and interpret two-way frequency tables of data when two categories are associated with each object being classified. Use the two-way table as a sample space to decide if events are independent and to approximate conditional probabilities. [S-CP4]

    Example: Collect data from a random sample of students in your school on their favorite subject among mathematics, science, and English. Estimate the probability that a randomly selected student from your school will favor science given that the student is in tenth grade. Do the same for other subjects and compare the results.

46. Recognize and explain the concepts of conditional probability and independence in everyday language and everyday situations. [S-CP5]

    Example: Compare the chance of having lung cancer if you are a smoker with the chance of being a smoker if you have lung cancer.

LO 3.2.2	use large datasets to explore and discover information and knowledge

Use the rules of probability to compute probabilities of compound events in a uniform probability model.

47. Find the conditional probability of *A* given *B* as the fraction of *B*'s outcomes that also belong to *A*, and interpret the answer in terms of the model. [S-CP6]

48. Apply the Addition Rule, $P(A \text{ or } B) = P(A) + P(B) - P(A \text{ and } B)$, and interpret the answer in terms of the model. [S-CP7]

49. (+) Apply the general Multiplication Rule in a uniform probability model, $P(A \text{ and } B) = P(A)P(B|A) = P(B)P(A|B)$, and interpret the answer in terms of the model. [S-CP8]

50. (+) Use permutations and combinations to compute probabilities of compound events and solve problems. [S-CP9]

DISCRETE MATHEMATICS

Discrete Mathematics is a course designed for students who have successfully completed the Algebra II With Trigonometry course and who choose not to continue mathematics study in the Precalculus or Analytical Mathematics courses. This course may be offered as an elective for students who have completed the four mathematics requirements for graduation.

Discrete Mathematics expands upon the topics of matrices, combinational reasoning, counting techniques, algorithms, sequences, series, and their applications. Students are expected to work in both individual and group settings to apply problem-solving strategies and to incorporate technological tools that extend beyond traditional instructional practices. The prerequisites for this course are Algebra I, Geometry, and Algebra II With Trigonometry.

Students will:

NUMBER AND QUANTITY

1. Analyze topics from elementary number theory, including perfect numbers and prime numbers, to determine properties of integers.

LO 1.1.3	analyze computational artifacts
LO 4.2.1	Express an algorithm in a language
LO 4.4.1	evaluate algorithmic analytically and empirically
LO 6.3.1	connect the concern of cybersecurity with the Internet and systems built on it
LO 7.3.1	analyze the beneficial and harmful effects of computing

2. Determine characteristics of sequences, including the Fibonacci sequence, the triangular numbers, and pentagonal numbers.
   Example: Write a sequence of the first 10 triangular numbers and hypothesize a formula to find the n^{th} triangular number.

LO 5.3.1	evaluate a program for correctness
LO 5.4.1	employ appropriate mathematical and logical concepts in programming

3. Use the recursive process and difference equations to create fractals, population growth models, sequences, series, and compound interest models.

LO 5.3.1	evaluate a program for correctness
LO 5.4.1	employ appropriate mathematical and logical concepts in programming

4. Convert between base ten and other bases.

ALGEBRA

5. Determine results of operations upon 3 × 3 and larger matrices, including matrix addition and multiplication of a matrix by a matrix, vector, or scalar.

6. Analyze determinants and inverses of 2 × 2, 3 × 3, and larger matrices to determine the nature of the solution set of the corresponding system of equations, including solving systems of equations in three variables by echelon row reduction and matrix inverse.

7. Solve problems through investigation and application of existence and nonexistence of Euler paths, Euler circuits, Hamilton paths, and Hamilton circuits.
   Example: Show why a 5 × 5 grid has no Hamilton circuit.

   a. Develop optimal solutions of application-based problems using existing and student-created algorithms.

LO 4.1.1	develop an algorithm designed to be implemented to run on a computer
LO 4.3.1	appropriately connect problems and potential algorithmic solutions.

8. Apply algorithms, including Kruskal's and Prim's, relating to minimum weight spanning trees, networks, flows, and Steiner trees.
   a. Use shortest path techniques to find optimal shipping routes.

LO 4.1.1	develop an algorithm designed to be implemented to run on a computer
LO 4.3.1	appropriately connect problems and potential algorithmic solutions.
LO 5.1.1	explain how programs implement algorithms

9. Determine a minimum project time using algorithms to schedule tasks in order, including critical path analysis, the list-processing algorithm, and student-created algorithms.

LO 5.2.1	use abstraction to manage complexity in programs

GEOMETRY

10. Use vertex-coloring techniques and matching techniques to solve application-based problems.

    Example: Use graph-coloring techniques to color a map of the western states of the United States so no adjacent states are the same color, including determining the minimum number of colors needed and why no fewer colors may be used.

LO 3.2.1	use computing to facilitate exploration and the discovery of connections in information
LO 3.2.2	use large datasets to explore and discover information and knowledge

11. Solve application-based logic problems using Venn diagrams, truth tables, and matrices.

LO 5.4.1	employ appropriate mathematical and logical concepts in programming

STATISTICS AND PROBABILITY

12. Use combinatorial reasoning and counting techniques to solve application-based problems.

    Example: Determine the probability of a safe opening on the first attempt given the combination uses the digits 2, 4, 6, and 8 with the order unknown.

    Answer: The probability of the safe opening on the first attempt is $\frac{1}{24}$.

LO 3.1.2	collaborate when processing information to gain insight and knowledge
LO 5.3.2	develop a correct program.

13. Analyze election data to compare election methods and voting apportionment, including determining strength within specific groups.

LO 3.1.1	use computers to process information to gain insight and knowledge
LO 3.1.2	collaborate when processing information to gain insight and knowledge
LO 3.1.3	communicate insight and knowledge gained from using computer programs to process information
LO 3.2.2	use large datasets to explore and discover information and knowledge

MATHEMATICAL INVESTIGATIONS

Mathematical Investigations is a course designed for students who have successfully completed the Algebra II With Trigonometry course and who choose not to continue mathematics study in the Precalculus or Analytical Mathematics courses. This course may be offered as an elective for students who have completed the four mathematics requirements for graduation.

Mathematical Investigations is intended to extend students' knowledge of mathematical development. Beginning with ancient numeration systems, students explore relationships between mathematics and nature, music, art, and architecture as well as the contributions of well-known mathematicians. It extends the scope of prerequisite courses, integrating topics with an emphasis on application-based problem solving. The wide range of topics and applied problems may lend itself to organizing the content into thematic units. The prerequisites for this course are Algebra I, Geometry, and Algebra II With Trigonometry.

Students will:

NUMBER AND QUANTITY

1. Critique ancient numeration systems and applications, including astronomy and the development and use of money and calendars.
   a. Determine relationships among mathematical achievements of ancient peoples, including the Sumerians, Babylonians, Egyptians, Mesopotamians, Chinese, Aztecs, and Incas.
   b. Explain origins of the Hindu-Arabic numeration system.
      Example: Perform addition and subtraction in both the Hindu-Arabic and the Roman numeration systems to compare place value and place holders.

2. Analyze mathematical relationships in music to interpret frequencies of musical notes and to compare mathematical structures of various musical instruments.
   Examples: Compare frequencies of notes exactly one octave apart on the musical scale; using frequencies and wave patterns of middle C, E above middle C, and G above middle C to explain why the C major chord is harmonious.

LO 1.1.1	use computing tools and techniques to create artifacts
LO 1.1.3	analyze computational artifacts
LO 7.2.1	connect computing with innovations in other fields

   a. Determine lengths of strings necessary to produce harmonic tones as in Pythagorean tuning.

3. Use special numbers, including e, i, π, and the golden ratio, to solve application-based problems.
   a. Identify transcendental numbers.
      Example: Calculate e to ten decimal places using a summation with $\frac{1}{n!}$.

4. Explain the development and uses of sets of numbers, including complex, real, rational, irrational, integer, whole, and natural numbers.
   a. Analyze contributions to the number system by well-known mathematicians, including Archimedes, John Napier, René Descartes, Sir Isaac Newton, Johann Carl Friedrich Gauss, and Julius Wilhelm Richard Dedekind.

      Example: Plot solutions to the polynomial equation, $x^2 - 6x + 11 = 0$, on the Gaussian plane.

ALGEBRA

5. Identify beginnings of algebraic symbolism and structure through the works of European mathematicians.
    a. Create a Fibonacci sequence when given two initial integers.

LO 5.3.1	evaluate a program for correctness
LO 7.2.1	connect computing with innovations in other fields

    b. Investigate Tartaglia's formula for solving cubic equations.

6. Explain the development and applications of logarithms, including contributions of John Napier, Henry Briggs, and the Bernoulli family.

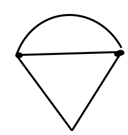
LO 4.4.1	evaluate algorithmic analytically and empirically

7. Justify the historical significance of the development of multiple perspectives in mathematics.
    Example: Relate the historical development of multiple perspectives to the works of Sir Isaac Newton and Gottfried Wilhelm von Leibniz in the foundations of calculus.

    a. Summarize the significance of René Descartes' Cartesian coordinate system.
    b. Interpret the foundation of analytic geometry with regard to geometric curves and algebraic relationships.

GEOMETRY

8. Solve problems from non-Euclidean geometry, including graph theory, networks, topology, and fractals.
    Examples: Observe the figure to the right to determine if it is traversable, and if it is, describe a path that will traverse it.
    Verify that two objects are topologically equivalent.
    Sketch four iterations of Sierpínski's triangle.



LO 4.2.1	Express an algorithm in a language
LO 6.2.1	explain characteristics of the Internet and the systems built on it.

9. Analyze works of visual art and architecture for mathematical relationships.
    Examples: Use Leonardo da Vinci's *Vitruvian Man* to explore the golden ratio.
    Identify mathematical patterns in Maurits Cornelis Escher's drawings, including the use of tessellations in art, quilting, paintings, pottery, and architecture.

    a. Summarize the historical development of perspective in art and architecture.

LO 7.2.1	connect computing with innovations in other fields

10. Determine the mathematical impact of the ancient Greeks, including Archimedes, Eratosthenes, Euclid, Hypatia, Pythagoras, and the Pythagorean Society.
    Example: Use Euclid's proposition to inscribe a regular hexagon within a circle.

    a. Construct multiple proofs of the Pythagorean Theorem.
    b. Solve problems involving figurate numbers, including triangular and pentagonal numbers.
       Example: Write a sequence of the first 10 triangular numbers and hypothesize a formula for finding the n^{th} triangular number.

11. Describe the development of mathematical tools and their applications.
    Examples: Use knotted ropes for counting; Napier's bones for multiplication; a slide rule for multiplying and calculating values of trigonometric, exponential, and logarithmic functions; and a graphing calculator for analyzing functions graphically and numerically.

LO 3.1.1	use computers to process information to gain insight and knowledge
LO 7.4.1	connect computing within economic, social, and cultural contexts

STATISTICS AND PROBABILITY

12. Summarize the history of probability, including the works of Blaise Pascal; Pierre de Fermat; Abraham de Moivre; and Pierre-Simon, marquis de Laplace.
    Example: Discuss the impact of probability on gaming, economics, and insurance.

LO 6.1.1	explain the abstractions in the Internet and how the Internet functions.

LO 3.1.1	use computers to process information to gain insight and knowledge
LO 7.4.1	connect computing within economic, social, and cultural contexts

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PRECALCULUS

Precalculus is a course designed for students who have successfully completed the Algebra II With Trigonometry course. This course is considered to be a prerequisite for success in calculus and college mathematics. Algebraic, graphical, numerical, and verbal analyses are incorporated during investigations of the Precalculus content standards. Parametric equations, polar relations, vector operations, and limits are introduced. Content for this course also includes an expanded study of polynomial and rational functions, conic sections, trigonometric functions, and logarithmic and exponential functions.

Application-based problem solving is an integral part of the course. Instruction should include appropriate use of technology to facilitate continued development of students' higher-order thinking skills.

Students will:

NUMBER AND QUANTITY

The Complex Number System

Represent complex numbers and their operations on the complex plane.

1. (+) Represent complex numbers on the complex plane in rectangular and polar form (including real and imaginary numbers), and explain why the rectangular and polar forms of a given complex number represent the same number. [N-CN4]

2. (+) Represent addition, subtraction, multiplication, and conjugation of complex numbers geometrically on the complex plane; use properties of this representation for computation. [N-CN5]
   Example: $(-1 + \sqrt{3}\, i)^3 = 8$ because $(-1 + \sqrt{3}\, i)$ has modulus 2 and argument 120°.

3. (+) Calculate the distance between numbers in the complex plane as the modulus of the difference, and the midpoint of a segment as the average of the numbers at its endpoints. [N-CN6]

Limits

Understand limits of functions.

4. Determine numerically, algebraically, and graphically the limits of functions at specific values and at infinity.
   a. Apply limits in problems involving convergence and divergence.

Vector and Matrix Quantities

Represent and model with vector quantities.

5. (+) Recognize vector quantities as having both magnitude and direction. Represent vector quantities by directed line segments, and use appropriate symbols for vectors and their magnitudes (e.g., **v**, |**v**|, ||**v**||, v). [N-VM1]

6. (+) Find the components of a vector by subtracting the coordinates of an initial point from the coordinates of a terminal point. [N-VM2]

7. (+) Solve problems involving velocity and other quantities that can be represented by vectors. [N-VM3]

Perform operations on vectors.

8. (+) Add and subtract vectors. [N-VM4]
   a. (+) Add vectors end-to-end, component-wise, and by the parallelogram rule. Understand that the magnitude of a sum of two vectors is typically not the sum of the magnitudes. [N-VM4a]
   b. (+) Given two vectors in magnitude and direction form, determine the magnitude and direction of their sum. [N-VM4b]
   c. (+) Understand vector subtraction **v** – **w** as **v** + (–**w**), where –**w** is the additive inverse of **w**, with the same magnitude as **w** and pointing in the opposite direction. Represent vector subtraction graphically by connecting the tips in the appropriate order, and perform vector subtraction component-wise. [N-VM4c]

9. (+) Multiply a vector by a scalar. [N-VM5]
   a. (+) Represent scalar multiplication graphically by scaling vectors and possibly reversing their direction; perform scalar multiplication component-wise, e.g., as $c(v_x, v_y) = (cv_x, cv_y)$. [N-VM5a]
   b. (+) Compute the magnitude of a scalar multiple *c***v** using ||*c***v**|| = |*c*|v. Compute the direction of *c***v** knowing that when |*c*|v ≠ 0, the direction of *c***v** is either along **v** (for $c > 0$) or against **v** (for $c < 0$). [N-VM5b]

Perform operations on matrices and use matrices in applications.

10. (+) Multiply a vector (regarded as a matrix with one column) by a matrix of suitable dimensions to produce another vector. Work with matrices as transformations of vectors. [N-VM11]

11. (+) Work with 2 × 2 matrices as transformations of the plane, and interpret the absolute value of the determinant in terms of area. [N-VM12]

PRECALCULUS

ALGEBRA

Seeing Structure in Expressions

Write expressions in equivalent forms to solve problems.

12. Derive the formula for the sum of a finite geometric series (when the common ratio is not 1), and use the formula to solve problems.* *(Extend to infinite geometric series.)* [A-SSE4]
    Example: Calculate mortgage payments.

LO 7.4.1	connect computing within economic, social, and cultural contexts

Arithmetic With Polynomials and Rational Expressions

Use polynomial identities to solve problems.

13. (+) Know and apply the Binomial Theorem for the expansion of $(x + y)^n$ in powers of x and y for a positive integer n, where x and y are any numbers, with coefficients determined, for example, by Pascal's Triangle. (The Binomial Theorem can be proved by mathematical induction or by a combinatorial argument.) [A-APR5]

Reasoning With Equations and Inequalities

Solve systems of equations.

14. (+) Represent a system of linear equations as a single matrix equation in a vector variable. [A-REI8]

FUNCTIONS

Conic Sections

Understand the graphs and equations of conic sections.

15. Create graphs of conic sections, including parabolas, hyperbolas, ellipses, circles, and degenerate conics, from second-degree equations.
    Example: Graph $x^2 - 6x + y^2 - 12y + 41 = 0$ or $y^2 - 4x + 2y + 5 = 0$.

    a. Formulate equations of conic sections from their determining characteristics.
       Example: Write the equation of an ellipse with center (5, -3), a horizontal major axis of length 10, and a minor axis of length 4.
       Answer: $\frac{(x-5)^2}{25} + \frac{(y+3)^2}{4} = 1.$

Interpreting Functions

Interpret functions that arise in applications in terms of the context. ***(Emphasize selection of appropriate models. Understand limits of functions.)***

16. For a function that models a relationship between two quantities, interpret key features of graphs and tables in terms of the quantities, and sketch graphs showing key features given a verbal description of the relationship. (*Key features include intercepts; intervals where the function is increasing, decreasing, positive, or negative; relative maximums and minimums; symmetries; end behavior; and periodicity. Determine odd, even, neither.)** [F-IF4]

17. Calculate and interpret the average rate of change of a function (presented symbolically or as a table) over a specified interval. Estimate the rate of change from a graph.* [F-IF6]

Analyze functions using different representations. ***(Focus on using key features to guide selection of appropriate type of model function with emphasis on piecewise, step, and absolute value. Also emphasize inverse and transformations of polynomials, rational, radical, absolute value, and trigonometric functions.)***

18. Graph functions expressed symbolically, and show key features of the graph, by hand in simple cases and using technology for more complicated cases.* [F-IF7]
    a. Graph square root, cube root, and piecewise-defined functions, including step functions and absolute value functions. [F-IF7b]
    b. Graph polynomial functions, identifying zeros when suitable factorizations are available, and showing end behavior. [F-IF7c]
    c. (+) Graph rational functions, identifying zeros and asymptotes when suitable factorizations are available, and showing end behavior. [F-IF7d]
    d. Graph exponential and logarithmic functions, showing intercepts and end behavior, and trigonometric functions, showing period, midline, and amplitude. [F-IF7e]

Building Functions

Build a function that models a relationship between two quantities.

19. (+) Compose functions. [F-BF1c]
    Example: If $T(y)$ is the temperature in the atmosphere as a function of height, and $h(t)$ is the height of a weather balloon as a function of time, then $T(h(t))$ is the temperature at the location of the weather balloon as a function of time.

Build new functions from existing functions.

20. Determine the inverse of a function and a relation.

21. (+) Verify by composition that one function is the inverse of another. [F-BF4b]

22. (+) Read values of an inverse function from a graph or a table, given that the function has an inverse. [F-BF4c]

23. (+) Produce an invertible function from a non-invertible function by restricting the domain. [F-BF4d]

24. (+) Understand the inverse relationship between exponents and logarithms, and use this relationship to solve problems involving logarithms and exponents. [F-BF5]

25. Compare effects of parameter changes on graphs of transcendental functions.
    Example: Explain the relationship of the graph $y = e^{x-2}$ to the graph $y = e^x$.

Trigonometric Functions

Recognize attributes of trigonometric functions and solve problems involving trigonometry.

26. Determine the amplitude, period, phase shift, domain, and range of trigonometric functions and their inverses.

27. Use the sum, difference, and half-angle identities to find the exact value of a trigonometric function.

28. Utilize parametric equations by graphing and by converting to rectangular form.
    a. Solve application-based problems involving parametric equations.
    b. Solve applied problems that include sequences with recurrence relations.

Extend the domain of trigonometric functions using the unit circle.

29. (+) Use special triangles to determine geometrically the values of sine, cosine, and tangent for $\frac{\pi}{3}$, $\frac{\pi}{4}$, and $\frac{\pi}{6}$, and use the unit circle to express the values of sine, cosine, and tangent for $\pi - x$, $\pi + x$, and $2\pi - x$ in terms of their values for x, where x is any real number. [F-TF3]

30. (+) Use the unit circle to explain symmetry (odd and even) and periodicity of trigonometric functions. [F-TF4]

Model periodic phenomena with trigonometric functions.

31. (+) Understand that restricting a trigonometric function to a domain on which it is always increasing or always decreasing allows its inverse to be constructed. [F-TF6]

32. (+) Use inverse functions to solve trigonometric equations that arise in modeling contexts; evaluate the solutions using technology, and interpret them in terms of the context.* [F-TF7]

Prove and apply trigonometric identities.

33. Prove the Pythagorean identity $\sin^2(\theta) + \cos^2(\theta) = 1$, and use it to find $\sin(\theta)$, $\cos(\theta)$, or $\tan(\theta)$ given $\sin(\theta)$, $\cos(\theta)$, or $\tan(\theta)$ and the quadrant of the angle. [F-TF8]

34. (+) Prove the addition and subtraction formulas for sine, cosine, and tangent, and use them to solve problems. [F-TF9]

GEOMETRY

Similarity, Right Triangles, and Trigonometry

Apply trigonometry to general triangles.

35. (+) Derive the formula $A = (\frac{1}{2})ab \sin(C)$ for the area of a triangle by drawing an auxiliary line from a vertex perpendicular to the opposite side. (*Apply formulas previously derived in Geometry.*) [G-SRT9]

Expressing Geometric Properties With Equations

Translate between the geometric description and the equation for a conic section.

36. (+) Derive the equations of a parabola given a focus and directrix. [G-GPE2]

37. (+) Derive the equations of ellipses and hyperbolas given the foci, using the fact that the sum or difference of distances from the foci is constant. [G-GPE3]

Explain volume formulas and use them to solve problems.

38. (+) Give an informal argument using Cavalieri's principle for the formulas for the volume of a sphere and other solid figures. [G-GMD2]

STATISTICS AND PROBABILITY

Interpreting Categorical and Quantitative Data

Summarize, represent, and interpret data on a single count or measurement variable.

39. Use statistics appropriate to the shape of the data distribution to compare center (median, mean) and spread (interquartile range, standard deviation) of two or more different data sets. *(Focus on increasing rigor using standard deviation.)* [S-ID2]

LO 2.3.1	use models and simulations to raise and answer questions
LO 3.1.1	use computers to process information to gain insight and knowledge
LO 3.1.2	collaborate when processing information to gain insight and knowledge
LO 3.1.3	communicate insight and knowledge gained from using computer programs to process information
LO 3.2.1	use computing to facilitate exploration and the discovery of connections in information
LO 3.2.2	use large datasets to explore and discover information and knowledge
LO 3.3.1	analyze the considerations involved in the computational manipulation of information

40. Interpret differences in shape, center, and spread in the context of the data sets, accounting for possible effects of extreme data points (outliers). *(Identify uniform, skewed, and normal distributions in a set of data. Determine the quartiles and interquartile range for a set of data.)* [S-ID3]

LO 2.3.1	use models and simulations to raise and answer questions
LO 3.1.1	use computers to process information to gain insight and knowledge
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LO 3.2.2	use large datasets to explore and discover information and knowledge
LO 3.3.1	analyze the considerations involved in the computational manipulation of information

41. Use the mean and standard deviation of a data set to fit it to a normal distribution and to estimate population percentages. Recognize that there are data sets for which such a procedure is not appropriate. Use calculators, spreadsheets, and tables to estimate areas under the normal curve. [S-ID4]

LO 2.3.1	use models and simulations to raise and answer questions
LO 3.1.1	use computers to process information to gain insight and knowledge
LO 3.1.2	collaborate when processing information to gain insight and knowledge
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LO 3.2.2	use large datasets to explore and discover information and knowledge
LO 3.3.1	analyze the considerations involved in the computational manipulation of information

Interpret linear models.

42. Compute (using technology) and interpret the correlation coefficient of a linear fit. [S-ID8]

LO 2.3.1	use models and simulations to raise and answer questions
LO 3.1.1	use computers to process information to gain insight and knowledge
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LO 3.2.1	use computing to facilitate exploration and the discovery of connections in information
LO 3.2.2	use large datasets to explore and discover information and knowledge
LO 3.3.1	analyze the considerations involved in the computational manipulation of information

43. Distinguish between correlation and causation. [S-ID9]

LO 2.3.1	use models and simulations to raise and answer questions
LO 3.1.1	use computers to process information to gain insight and knowledge
LO 3.1.2	collaborate when processing information to gain insight and knowledge
LO 3.1.3	communicate insight and knowledge gained from using computer programs to process information
LO 3.2.1	use computing to facilitate exploration and the discovery of connections in information
LO 3.2.2	use large datasets to explore and discover information and knowledge
LO 3.3.1	analyze the considerations involved in the computational manipulation of information

Making Inferences and Justifying Conclusions

Understand and evaluate random processes underlying statistical experiments.

44. Understand statistics as a process for making inferences about population parameters based on a random sample from that population. [S-IC1]

LO 3.1.2	collaborate when processing information to gain insight and knowledge

45. Decide if a specified model is consistent with results from a given data-generating process, e.g., using simulation. [S-IC2]
    Example: A model says a spinning coin falls heads up with probability 0.5. Would a result of 5 tails in a row cause you to question the model?

LO 3.3.1	analyze the considerations involved in the computational manipulation of information

Make inferences and justify conclusions from sample surveys, experiments, and observational studies.

46. Recognize the purposes of and differences among sample surveys, experiments, and observational studies; explain how randomization relates to each. [S-IC3]

LO 3.1.1	use computers to process information to gain insight and knowledge

47. Use data from a sample survey to estimate a population mean or proportion; develop a margin of error through the use of simulation models for random sampling. [S-IC4]

LO 3.2.2	use large datasets to explore and discover information and knowledge

48. Use data from a randomized experiment to compare two treatments; use simulations to decide if differences between parameters are significant. [S-IC5]

LO 3.2.2	use large datasets to explore and discover information and knowledge

49. Evaluate reports based on data. [S-IC6]

LO 3.3.1	analyze the considerations involved in the computational manipulation of information

Using Probability to Make Decisions

Calculate expected values and use them to solve problems.

50. (+) Define a random variable for a quantity of interest by assigning a numerical value to each event in a sample space; graph the corresponding probability distribution using the same graphical displays as for data distributions. [S-MD1]

51. (+) Calculate the expected value of a random variable; interpret it as the mean of the probability distribution. [S-MD2]

52. (+) Develop a probability distribution for a random variable defined for a sample space in which theoretical probabilities can be calculated; find the expected value. [S-MD3]
    Example: Find the theoretical probability distribution for the number of correct answers obtained by guessing on all five questions of a multiple-choice test where each

question has four choices, and find the expected grade under various grading schemes.

53. (+) Develop a probability distribution for a random variable defined for a sample space in which probabilities are assigned empirically; find the expected value. [S-MD4]
    Example: Find a current data distribution on the number of television sets per household in the United States, and calculate the expected number of sets per household. How many television sets would you expect to find in 100 randomly selected households?

LO 3.1.1	use computers to process information to gain knowledge and insight

Use probability to evaluate outcomes of decisions.

54. (+) Weigh the possible outcomes of a decision by assigning probabilities to payoff values and finding expected values. [S-MD5]
    a. Find the expected payoff for a game of chance. [S-MD5a]
        Examples: Find the expected winnings from a state lottery ticket or a game at a fast-food restaurant.
    b. Evaluate and compare strategies on the basis of expected values. [S-MD5b]
        Example: Compare a high-deductible versus a low-deductible automobile insurance policy using various, but reasonable, chances of having a minor or a major accident.

LO 3.3.1	analyze the considerations involved in the computational manipulation of information

ANALYTICAL MATHEMATICS

Analytical Mathematics is a course designed for students who have successfully completed the Algebra II With Trigonometry course. It is considered to be parallel in rigor to Precalculus. This course provides a structured introduction to important areas of emphasis in most postsecondary studies that pursue a concentration in mathematics. Linear algebra, logic, vectors, and matrices are topics that are given more in-depth coverage than in previous courses. Application-based problem solving is an integral part of this course. To assist students with numerical and graphical analysis, the use of advanced technological tools is highly recommended.

While this course may be taken either prior to or after Precalculus, it is recommended that students who are interested in postsecondary studies in engineering successfully complete the Precalculus course as well as, where available, an Advanced Placement or International Baccalaureate calculus course.

Students will:

NUMBER AND QUANTITY

Vector and Matrix Quantities

Represent and model with vector quantities.

1. (+) Recognize vector quantities as having both magnitude and direction. Represent vector quantities by directed line segments, and use appropriate symbols for vectors and their magnitudes (e.g., $\mathbf{v}$, $|\mathbf{v}|$, $||\mathbf{v}||$), including the use of eigen-values and eigen-vectors. [N-VM1]

2. (+) Solve problems involving velocity and other quantities that can be represented by vectors, including navigation (e.g., airplane, aerospace, oceanic). [N-VM3]

3. (+) Add vectors end-to-end, component-wise, and by the parallelogram rule. Understand that the magnitude of a sum of two vectors is typically not the sum of the magnitudes. Find the dot product and the cross product of vectors. [N-VM4a]

4. (+) Given two vectors in magnitude and direction form, determine the magnitude and direction of their sum, including vectors in complex vector spaces. [N-VM4b]

5. (+) Understand vector subtraction $\mathbf{v} - \mathbf{w}$ as $\mathbf{v} + (-\mathbf{w})$, where $(-\mathbf{w})$ is the additive inverse of $\mathbf{w}$, with the same magnitude as $\mathbf{w}$ and pointing in the opposite direction. Represent vector subtraction graphically by connecting the tips in the appropriate order, and perform vector subtraction component-wise, including vectors in complex vector spaces. [N-VM4c]

Perform operations on matrices and use matrices in applications.

6. (+) Use matrices to represent and manipulate data, e.g., to represent payoffs or incidence relationships in a network, including linear programming. [N-VM6]

7. (+) Multiply matrices by scalars to produce new matrices, e.g., as when all of the payoffs in a game are doubled, including rotation matrices. [N-VM7]

8. (+) Understand that the zero and identity matrices play a role in matrix addition and multiplication similar to the role of 0 and 1 in the real numbers. The determinant of a square matrix is nonzero if and only if the matrix has a multiplicative inverse. Solve matrix equations using augmented matrices. [N-VM10]

9. (+) Multiply a vector (regarded as a matrix with one column) by a matrix of suitable dimensions to produce another vector. Work with matrices as transformations of vectors, including matrices larger than 2 × 2. [N-VM11]

10. (+) Work with 2 × 2 matrices as transformations of the plane, and interpret the absolute value of the determinant in terms of area. Solve matrix application problems using reduced row echelon form. [N-VM12]

Complex Numbers

Use complex numbers in polynomial identities and equations.

11. (+) Know the Fundamental Theorem of Algebra; show that it is true for quadratic polynomials. Understand the importance of using complex numbers in graphing functions on the Cartesian or complex plane. [N-CN9]

Limits

Understand limits of functions.

12. Calculate the limit of a sequence, of a function, and of an infinite series.

ALGEBRA

Seeing Structure in Expressions

13. Use the laws of Boolean Algebra to describe true/false circuits. Simplify Boolean expressions using the relationships between conjunction, disjunction, and negation operations.

LO 5.4.1	employ appropriate mathematical and logical concepts in programming

14. Use logic symbols to write truth tables.

LO 5.4.1	employ appropriate mathematical and logical concepts in programming

Arithmetic With Polynomials and Rational Functions

15. Reduce the degree of either the numerator or denominator of a rational function by using partial fraction decomposition or partial fraction expansion.

FUNCTIONS

Trigonometric Functions

Extend the domain of trigonometric functions using the unit circle.

16. (+) Use the unit circle to explain symmetry (odd and even) and periodicity of trigonometric functions. [F-TF4].

Apply trigonometry to general triangles.

17. (+) Prove the Law of Sines and the Law of Cosines and use them to solve problems. Understand Law of Sines = 2*r*, where *r* is the radius of the circumscribed circle of the triangle. Apply the Law of Tangents. [G-SRT10]

18. Apply Euler's and deMoivre's formulas as links between complex numbers and trigonometry.