ARKANSAS ACADEMIC STANDARDS

Arkansas Mathematics Standards

Pre-Calculus
Content Standards
2016

Course Title: Pre-Calculus
Course/Unit Credit: 1
Course Number: 433000
Teacher Licensure: Please refer to the Course Code Management System (https://adedata.arkansas.gov/ccms/) for the most current licensure codes.
Grades: 9-12
Prerequisites: Algebra I, Geometry, Algebra II

Pre-Calculus

Pre-Calculus will emphasize a study of trigonometric functions and identities as well as applications of right triangle trigonometry and circular functions. Students will use symbolic reasoning and analytical methods to represent mathematical situations, express generalizations, and study mathematical concepts and the relationships among them. Students will use functions and equations as tools for expressing generalizations. Pre-Calculus does not require Arkansas Department of Education approval.

Strand	Content Standard
Number and Quantity	
	1. Students will use complex numbers and determine how polar and rectangular coordinates are related.
	2. Students will perform operations with vectors and use those skills to solve problems.
Trigonometry	
	3. Students will develop and apply the definitions of the six trigonometric functions and use the definitions to solve problems and verify identities.
	4. Students will solve trigonometric equations and sketch the graph of periodic trigonometric functions.
Conic Sections	
	5. Students will identify, analyze, and sketch the graphs of the conic sections and relate their equations and graphs.
Functions	
	6. Students will be able to find the inverse of functions and use composition of functions to prove that two functions are inverses.
	7. Students will be able to interpret different types of functions and their key characteristics including polynomial, exponential, logarithmic, power, trigonometric, rational, and other types of functions.

Strand: Number and Quantity

Content Standard 1: Students will use complex numbers and determine how polar and rectangular coordinates are related.

NQ.1.PC.1	• Find the conjugate of a complex number • Use conjugates to find quotients of complex numbers • (+)Use conjugates to find moduli
NQ.1.PC.2	• (+) Represent complex numbers on the complex plane in rectangular and polar form (including real and imaginary numbers) • (+) Explain why the rectangular and polar forms of a given complex number represent the same number
NQ.1.PC.3	• (+) Represent addition, subtraction, multiplication, and conjugation of complex numbers geometrically on the complex plane • (+) Use properties of geometrical representation for computation For example: $(-1 + \sqrt{3}i)^3 = 8$ because $(-1 + \sqrt{3}i)$ has modulus 2 and argument 120°.
NQ.1.PC.4	(+) Calculate the distance between numbers in the complex plane as the modulus of the difference, and the midpoint of a segment as the average of the numbers at its endpoints

Strand: Number and Quantity

Content Standard 2: Students will perform operations with vectors and use those skills to solve problems.

NQ.2.PC.1	• (+) Recognize vector quantities as having both magnitude and direction • (+) Represent vector quantities by directed line segments, and use appropriate symbols for vectors and their magnitudes (e.g., ***v***, |***v***|, ||***v***||, *v*)
NQ.2.PC.2	(+) Find the components of a vector by subtracting the coordinates of an initial point from the coordinates of a terminal point
NQ.2.PC.3	Solve problems involving velocity and other quantities that can be represented by *vectors*
NQ.2.PC.4	(+) Add and subtract vectors: • Add vectors end-to-end, component-wise, and by the parallelogram rule. Understand that the magnitude of a sum of two vectors is typically not the sum of the magnitudes • Given two vectors in magnitude and direction form, determine the magnitude and direction of their sum • Understand vector subtraction ***v*** **-** ***w*** as ***v*** + (-***w***), where **-*w*** is the additive inverse of ***w***, with the same magnitude as ***w*** and pointing in the opposite direction • Represent vector subtraction graphically by connecting the tips in the appropriate order • Perform vector subtraction component-wise
NQ.2.PC.5	(+) Multiply a vector by a scalar: • Represent scalar multiplication graphically by scaling vectors and possibly reversing their direction • Perform scalar multiplication component-wise, e.g., as $c(v_x, v_y) = (cv_x, cv_y)$ • Compute the magnitude of a scalar multiple ***cv*** using ||*c**v***|| = |*c*|***v***. • Compute the direction of ***cv*** knowing that when |*c*|***v*** ≠ 0, the direction of ***cv*** is either along ***v*** (for $c > 0$) or against ***v*** (for $c < 0$)
NQ.2.PC.6	• (+) Multiply a vector (regarded as a matrix with one column) by a matrix of suitable dimensions to produce another vector • (+)Work with matrices as transformations of vectors

Strand: Trigonometry

Content Standard 3: Students will develop and apply the definitions of the six trigonometric functions and use the definitions to solve problems and verify identities.

T.3.PC.1	Understand radian measure of an angle as the length of the arc on the unit circle subtended by the angle
T.3.PC.2	Explain how the unit circle in the coordinate plane enables the extension of trigonometric functions to all real numbers, interpreted as radian measures of angles traversed around the unit circle
T.3.PC.3	• (+) Use special right triangles to determine geometrically the exact values of sine, cosine, tangent for $\frac{\pi}{3}, \frac{\pi}{4}, \frac{\pi}{6}$, and $\frac{\pi}{2}$ • (+) Use the unit circle to express the values of sine, cosine, and tangent for $\pi - x, \pi + x$, and $2\pi - x$ in terms of their exact values for x, where x is any real number
T.3.PC.4	• (+)Develop the Pythagorean identity, $\sin^2(\theta) + \cos^2(\theta) = 1$. • (+)Given $\sin(\theta)$, $\cos(\theta)$, or $\tan(\theta)$ and the quadrant of the angle, use the Pythagorean identity to find the remaining trigonometric functions
T.3.PC.5	(+) Develop the addition and subtraction formulas for sine, cosine, and tangent and use them to solve problems
T.3.PC.6	Derive the formula $A = \left(\frac{1}{2}\right) ab \sin C$ for the area of a triangle by drawing an auxiliary line from a vertex perpendicular to the opposite side
T.3.PC.7	Prove the *Law of Sines* and the *Law of Cosines* and use them to solve problems
T.3.PC.8	(+) Understand and apply the Law of Sines and the Law of Cosines to find unknown measurements in right and non-right triangles Note: Examples should include, but are not limited to surveying problems and problems related to resultant forces.
T.3.PC.9	Define and use reciprocal functions, cosecant, secant, and cotangent to solve problems

Strand: Trigonometry

Content Standard 4: Students will solve trigonometric equations and sketch the graph of periodic trigonometric functions.

T.4.PC.1	Use the unit circle to explain symmetry (odd and even) and periodicity of *trigonometric functions*
T.4.PC.2	Choose trigonometric functions to model periodic phenomena with specified amplitude, frequency, and midline*
T.4.PC.3	(+) Understand that restricting a trigonometric function to a domain on which it is always increasing or always decreasing allows its inverse to be constructed Note: Recognizing that the domain requires restriction because the function is not one-to-one, is acceptable for algebra 2. Whereas knowledge of how to restrict the domain and find the inverse is usually reserved for a fourth year mathematics course.
T.4.PC.4	(+) Use inverse functions to: • Solve trigonometric equations that arise in modeling context(s)* • Evaluate the solutions of trigonometric equations, with or without technology • Interpret the solutions of trigonometric equations in terms of the context(s)*
T.4.PC.5	Recognize that some trigonometric equations have infinitely many solutions and be able to state a general formula to represent the infinite solutions

Strand: Conic Sections

Content Standard 5: Students will identify, analyze, and sketch the graphs of the conic sections and relate their equations and graphs.

CS.5.PC.1	• Derive the equation of a circle of given center and radius using the Pythagorean Theorem • Complete the square to find the center and radius of a circle given by an equation Note: Students should also be able to identify the center and radius when given the equation of a circle and write the equation given a center and radius.
CS.5.PC.2	(+)Derive the equation of a parabola given a focus and directrix
CS.5.PC.3	(+) Derive the equations of ellipses and hyperbolas given the foci, using the fact that the sum or difference of distances from the foci is constant
CS.5.PC.4	Find the equations for the *asymptotes* of a hyperbola
CS.5.PC.5	Complete the square in order to generate an equivalent form of an equation for a conic section; use that equivalent form to identify key characteristics of the conic section
CS.5.PC.6	Identify, graph, write, and analyze equations of each type of conic section, using properties such as symmetry, intercepts, foci, *asymptotes*, and *eccentricity*, and using technology when appropriate
CS.5.PC.7	Solve systems of equations and inequalities involving conics and other types of equations, with and without appropriate technology

Strand: Functions

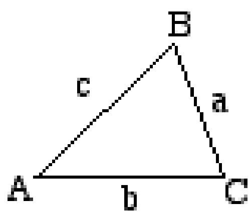
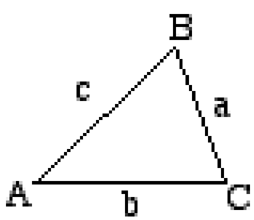
Content Standard 6: Students will be able to find the inverse of functions and use composition of functions to prove that two functions are inverses.

F.6.PC.1	Write a function that describes a relationship between two quantities*: • From a context, determine an explicit expression, a recursive process, or steps for calculation • Combine standard function types using arithmetic operations. (e.g., given that *f(x)* and *g(x)* are functions developed from a context, find *(f + g)(x), (f – g)(x), (fg)(x), (f/g)(x),* and any combination thereof, given $g(x) \neq 0$.) • Compose functions
F.6.PC.2	Find inverse functions: • Solve an equation of the form $y = f(x)$ for a simple function f that has an inverse and write an expression for the inverse For example, $f(x) = 2x^2$ or $f(x) = (x + 1)/(x - 1)$ for $x \neq 1$ • Verify by composition that one function is the inverse of another • Read values of an inverse function from a graph or a table, given that the function has an inverse • (+) Produce an invertible function from a non-invertible function by restricting the domain
F.6.PC.3	• Understand the inverse relationship between exponents and logarithms • Use the inverse relationship between exponents and logarithms to solve problems

Strand: Functions

Content Standard 7: Students will be able to interpret different types of functions and their key characteristics including polynomial, exponential, logarithmic, power, trigonometric, rational, and other types of functions.

F.7.PC.1	Recognize that sequences are functions, sometimes defined recursively, whose domain is a subset of the integers For example: The Fibonacci sequence is defined recursively by $(0) = (1) = 1$, $(n + 1) = (n) + (n - 1)$ for $n \geq 1$.
F.7.PC.2	Derive the formula for the sum of a finite geometric series (when the common ratio is not 1), and use the formula to solve problems For example: Calculate mortgage payments.
F.7.PC.3	(+) Know and apply the Binomial Theorem for the expansion of $(x + y)^n$ in powers of x and y for a positive integer n, where x and y are any numbers, with coefficients determined for example by Pascal's Triangle Note: The Binomial Theorem can be proved by mathematical induction or by a combinatorial argument.
F.7.PC.4	For a function that models a relationship between two quantities: • Interpret key features of graphs and tables in terms of the quantities, and • Sketch graphs showing key features given a verbal description of the relationship Note: Key features may include but not limited to: intercepts; intervals where the function is increasing, decreasing, positive, or negative; relative maximums and minimums; symmetries; end behavior; and periodicity.*
F.7.PC.5	• Calculate and interpret the average rate of change of a function (presented algebraically or as a table) over a specified interval* • Estimate the rate of change from a graph*
F.7.PC.6	Graph functions expressed algebraically and show key features of the graph, with and without technology: • Graph linear and quadratic functions and, when applicable, show intercepts, maxima, and minima • Graph square root, cube root, and piecewise-defined functions, including step functions and absolute value functions • Graph power and polynomial functions, identifying zeros when suitable factorizations are available, and showing end behavior • (+) Graph rational functions, identifying zeros and asymptotes when suitable factorizations are available, and showing end behavior • Graph exponential and logarithmic functions, showing intercepts and end behavior • (+) Graph trigonometric functions, showing period, midline, and amplitude
F.7.PC.7	Compare properties of two functions each represented in a different way (algebraically, graphically, numerically in tables, or by verbal descriptions)
F.7.PC.8	Build functions to model real-world applications using algebraic operations on functions and composition, with and without appropriate technology (e.g., profit functions as well as volume and surface area, optimization subject to constraints)

Amplitude	Half the difference between the minimum and maximum values of the range; only periodic functions with a bounded range have an amplitude
Asymptote(s)	Line(s) to which a graph becomes arbitrarily close as the value of x or y increases or decreases without bound (e.g., vertical, horizontal, slant)
Eccentricity	A number that indicates how drawn out or attenuated a conic section is; eccentricity is represented by the letter e (no relation to $\mathrm{e} = 2.718\ldots$)
Exponential Function(s)	Function(s) in which the variable(s) occurs in the exponent [e.g., $f(x) = ab^x, b > 0$]
Inverse Function(s)	Two functions f and g are inverse functions, if and only if both their compositions yield the identity function {e.g., $[f \circ g](x) = x$ and $[g \circ f](x) = x$}
Law of Cosines	An equation relating the cosine of an interior angle and the lengths of the sides of a triangle; the Pythagorean theorem is a corollary of the Law of Cosines $c^2 = a^2 + b^2 - 2ab\cos C$ $b^2 = a^2 + c^2 - 2ac\cos B$ $a^2 = b^2 + c^2 - 2bc\cos A$ 
Law of Sines	Equations relating the sines of the interior angles of a triangle and the corresponding opposite sides $\frac{\sin A}{a} = \frac{\sin B}{b} = \frac{\sin C}{c}$ 
Logarithmic Functions	Function of the form $y = \log_b x$, where $b > 0, x > 0$ and $b \neq 1$
Modulus (pl. Moduli)	For complex number(s) in polar form $z = \mathrm{r}(\cos\theta + i\sin\theta)$ the modulus is r
Phase Shift	Horizontal shift for a periodic function
Point Discontinuities	A point at which the graph of a relation or function is not connected

Polar Form(s) of a Complex Number	The polar form(s) or trigonometric form(s) of the complex number $z = a + bi$ is $z = \mathrm{r}(\cos\theta\ +\ i\sin\theta)$ where $a = r\cos\theta$, $b = r\sin\theta$, $r = \sqrt{a^2 + b^2}$, and $\tan\theta = b/a$
Scalar	Any real number, or any quantity that can be measured using a single real number; temperature, length, and mass are all scalars; a scalar is said to have magnitude but no direction
Trigonometric Function(s)	The six functions are sine, cosine, tangent, cosecant, secant, and cotangent
Vector(s)	Quantity or quantities with magnitude and direction in the plane or in space, defined by an ordered pair or triple of real numbers