Geometry

Content Standards

2016

Compiled using the Arkansas Mathematics Standards

Course Title: Geometry
Course/Unit Credit: 1
Course Number: 431000
Teacher Licensure: Please refer to the Course Code Management System (https://adedata.arkansas.gov/ccms/) for the most current licensure codes.
Grades: 9-12
Prerequisite: Algebra I or Algebra A/B

Course Description: "The fundamental purpose of the course in Geometry is to formalize and extend students' geometric experiences from the middle grades. Students explore more complex geometric situations and deepen their explanations of geometric relationships, moving towards formal mathematical arguments. Important differences exist between this Geometry course and the historical approach taken in Geometry classes. For example, transformations are emphasized early in this course. Close attention should be paid to the introductory content for the Geometry conceptual category found in the high school AMS.

This document was created to delineate the standards for this course in a format familiar to the educators of Arkansas. For the state-provided Algebra A/B, Algebra I, Geometry A/B, Geometry, and Algebra II documents, the language and structure of the Arkansas Mathematics Standards (ASM) have been maintained. The following information is helpful to correctly read and understand this document.

"**Standards** define what students should understand and be able to do.
Clusters are groups of related standards. Note that standards from different clusters may sometimes be closely related, because mathematics is a connected subject.
Domains are larger groups of related standards. Standards from different domains may sometimes be closely related." - http://www.corestandards.org/

Standards do not dictate curriculum or teaching methods. For example, just because topic A appears before topic B in the standards for a given grade, it does not necessarily mean that topic A must be taught before topic B. A teacher might prefer to teach topic B before topic A, or might choose to highlight connections by teaching topic A and topic B at the same time. Or, a teacher might prefer to teach a topic of his or her own choosing that leads, as a byproduct, to students reaching the standards for topics A and B.

Notes:

1. Teacher notes offer clarification of the standards.
2. The Plus Standards (+) from the Arkansas Mathematics Standards may be incorporated into the curriculum to adequately prepare students for more rigorous courses (e.g., Advanced Placement, International Baccalaureate, or concurrent credit courses).
3. Italicized words are defined in the glossary.
4. All items in a bulleted list must be taught.
5. Asterisks (*) identify potential opportunities to integrate content with the modeling practice.

The following abbreviations are for the conceptual categories and domains for the Arkansas Mathematics Standards. For example, the standard HSN.RN.B.3 is in the High School Number and Quantity conceptual category and in The Real Number System domain.

High School Number and Quantity – HSN
- The Real Number System – RN
- Quantities – Q
- The Complex Number System – CN
- Vectors and Matrix Quantities – VM

High School Algebra – HSA
- Seeing Structure in Expressions – SSE
- Arithmetic with Polynomials and Rational Expressions – APR
- Creating Equations – CED
- Reasoning with Equations and Inequalities – REI

High School Functions – HSF
- Interpreting Functions – IF
- Building Functions – BF
- Linear, Quadratic and Exponential Models – LE
- Trigonometric Functions – TF

High School Geometry – HSG
- Congruence – CO
- Similarity, Right Triangles, and Trigonometry – SRT
- Circles – C
- Expressing Geometric Properties with Equations – GPE
- Geometric Measurement and Dimension – GMD
- Modeling with Geometry – MG

High School Statistics and Probability – HSS
- Interpreting Categorical and Quantitative Data – ID
- Making Inferences and Justifying Conclusions – IC
- Conditional Probability and the Rules of Probability – CP
- Using Probability to Make Decisions – MD

Geometry

Domain	Cluster
Congruence	
	1. Investigate transformations in the plane
	2. Understand congruence in terms of rigid motions
	3. Apply and prove geometric theorems
	4. Make geometric constructions
	5. Logic and Reasoning
Similarity, Right Triangles, and Trigonometry	
	6. Understand similarity in terms of similarity transformations
	7. Apply and prove theorems involving similarity
	8. Define trigonometric ratios and solve problems involving right triangles
	9. Apply trigonometric to general triangles
Circles	
	10. Understand and apply theorems about circles
	11. Find arc lengths and areas of sectors of circles
Expressing Geometric Properties with Equations	
	12. Translate between the geometric description and the equation of a conic section
	13. Use coordinates to prove simple geometric theorems algebraically
Geometric measurement and dimension	
	14. Explain volume formulas and use them to solve problems
	15. Visualize relationships between two-dimensional and three-dimensional objects
Modeling with Geometry	
	16. Apply geometric concepts in modeling situations

Domain: Congruence

Cluster(s): 1. Investigate transformations in the plane
2. Understand congruence in terms of rigid motions
3. Apply and prove geometric theorems
4. Make geometric constructions
5. Logic and Reasoning

HSG.CO.A.1	1	Based on the undefined notions of *point*, *line*, *plane*, distance along a line, and distance around a circular arc, define: • *Angle* • *Line segment* • *Circle* • *Perpendicular lines* • *Parallel lines*
HSG.CO.A.2	1	• Represent *transformations* in the *plane* (e.g., using transparencies, tracing paper, geometry software) • Describe *transformations* as functions that take points in the *plane* as inputs and give other points as outputs • Compare *transformations* that preserve distance and angle to those that do not (e.g., *translation* versus *dilation*)
HSG.CO.A.3	1	Given a *rectangle, parallelogram, trapezoid*, or *regular polygon*, describe *the rotations* and *reflections* that carry it onto itself
HSG.CO.A.4	1	Develop definitions of *rotations, reflections*, and *translations* in terms of *angles, circles, perpendicular lines, parallel lines*, and *line segments*
HSG.CO.A.5	1	• Given a geometric figure and a *rotation, reflection,* or *translation*, draw the transformed figure (e.g., using graph paper, tracing paper, miras, geometry software) • Specify a sequence of *transformations* that will carry a given figure onto another
HSG.CO.B.6	2	• Use geometric descriptions of *rigid motions* to transform figures and to predict the effect of a given *rigid motion* on a given figure • Given two figures, use the definition of congruence in terms *of rigid motions* to decide if they are congruent
HSG.CO.B.7	2	Use the definition of congruence in terms of *rigid motions* to show that two triangles are *congruent* if and only if *corresponding* pairs of sides and *corresponding* pairs of angles are *congruent*
HSG.CO.B.8	2	Explain how the criteria for triangle congruence (ASA, SAS, and SSS) follow from the definition of congruence in terms of *rigid motions* Investigate congruence in terms of *rigid motion* to develop the criteria for triangle congruence (ASA, SAS, AAS, SSS, and HL) Note: The emphasis in this standard should be placed on investigation.
HSG.CO.C.9	3	Apply and prove *theorems* about lines and angles Note: Theorems include but are not limited to: *vertical angles* are congruent; when a *transversal* crosses *parallel lines*, *alternate interior angles* are congruent and *corresponding angles* are congruent; points on a *perpendicular bisector* of a *line segment* are exactly those equidistant from the segment's endpoints. Note: Proofs are not an isolated topic and therefore should be integrated throughout the course.

Domain: Congruence

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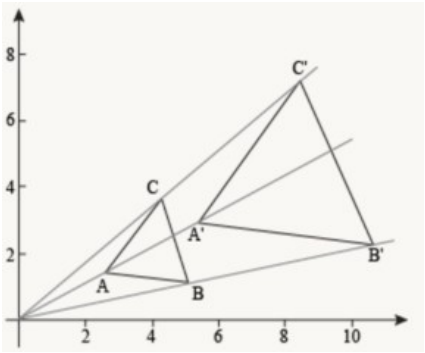
HSG.CO.C.10	3	Apply and prove *theorems* about triangles Note: Theorems include but are not limited to: measures of *interior angles* of a triangle sum to 180°; base angles of isosceles triangles are congruent; the segment joining midpoints of two sides of a triangle is parallel to the third side and half the length; the *medians* of a triangle meet at a point. Note: Proofs are not an isolated topic and therefore should be integrated throughout the course.
HSG.CO.C.11	3	Apply and prove *theorems* about quadrilaterals Note: Theorems include but are not limited to relationships among the sides, angles, and diagonals of quadrilaterals and the following theorems concerning *parallelograms*: opposite sides are congruent, opposite angles are congruent, the diagonals of a *parallelogram* bisect each other, and conversely, *rectangles are parallelograms* with congruent diagonals. Note: Proofs are not an isolated topic and therefore should be integrated throughout the course.
HSG.CO.D.12	4	Make formal geometric constructions with a variety of tools and methods (compass and straightedge, string, reflective devices, paper folding, dynamic geometric software) Note: Constructions may include but are not limited to: copying a segment; copying an angle; bisecting a segment; bisecting an angle; constructing *perpendicular lines*, including the *perpendicular bisector* of a *line segment*; and constructing a line parallel to a given line through a point not on the line. Note: Constructions are not an isolated topic and therefore should be integrated throughout the course.
HSG.CO.D.13	4	Construct an equilateral triangle, a square, and a regular hexagon *inscribed* in a circle Note: Constructions are not an isolated topic and therefore should be integrated throughout the course.
HSG.CO.E.14	5	Apply *inductive reasoning* and *deductive reasoning* for making predictions based on real world situations using: • *Conditional Statements* (*inverse, converse, and contrapositive*) • *Venn Diagrams* Note: This is not intended to be an isolated topic but instead to support concepts throughout the course.

Domain: Similarity, Right Triangles, and Trigonometry

Cluster(s): 6. Understand similarity in terms of similarity transformations
7. Apply and prove theorems involving similarity
8. Define trigonometric ratios and solve problems involving right triangles
9. Apply trigonometry to general triangles

HSG.SRT.A.1	6	Verify experimentally the properties of *dilations* given by a center and a *scale factor* • A *dilation* takes a line not passing through the center of the *dilation* to a *parallel line*, and leaves a line passing through the center unchanged • The *dilation* of a *line segment* is longer or shorter in the ratio given by the *scale factor*  http://www.shmoop.com/common-core-standards/ccss-hs-g-srt-1a.html
HSG.SRT.A.2	6	Given two figures: • Use the definition of *similarity* in terms of similarity transformations to determine if they are *similar* • Explain using similarity transformations the meaning of similarity for triangles as the equality of all corresponding pairs of angles and the proportionality of all corresponding pairs of sides
HSG.SRT.A.3	6	Use the properties of similarity transformations to establish the AA~, SAS~, SSS~ criteria for two triangles to be similar
HSG.SRT.D.11	9	(+) Understand and apply the Law of Sines and the Law of Cosines to find unknown measurements in right and non-right triangles Note: Examples should include, but are not limited to surveying problems and problems related to resultant forces.

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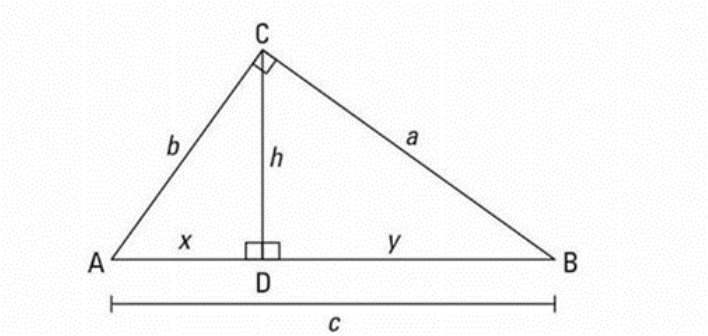
HSG.SRT.B.4	7	Use triangle similarity to apply and prove theorems about triangles Note: Theorems include: a line parallel to one side of a triangle divides the other two proportionally, and conversely; the Pythagorean Theorem proved using triangle similarity.  $\frac{x}{b}=\frac{b}{c}, \quad \frac{y=c-x}{a}=\frac{a}{c}$ $x=\frac{b^2}{c}, \quad c-x=\frac{a^2}{c}$ $x+(c-x)=c$ $\frac{a^2}{c}+\frac{b^2}{c}=c$ $a^2+b^2=c^2$
HSG.SRT.B.5	7	• Use congruence (SSS, SAS, ASA, AAS, and HL) and similarity (AA~, SSS~, SAS~) criteria for triangles to solve problems • Use congruence and similarity criteria to prove relationships in geometric figures

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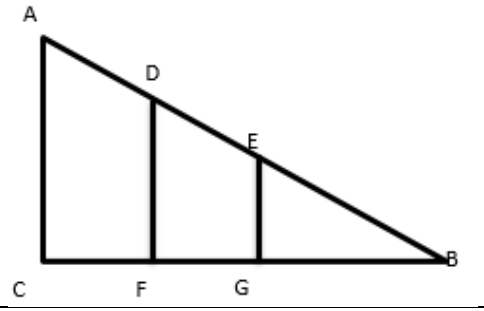
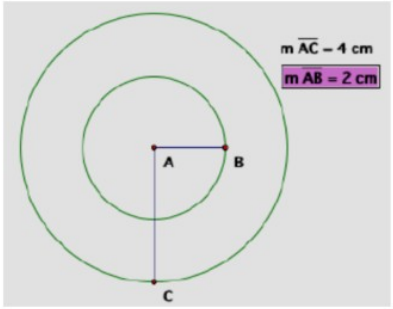
HSG.SRT.C.6	8	Understand that by similarity, side ratios in right triangles are properties of the angles in the triangle, leading to definitions of trigonometric ratios for acute angles For example: Trigonometric ratios are related to the acute angles of a triangle, not the right angle. The values of the trigonometric ratio depend only on the angle. Consider the following three similar right triangles, why are they similar? 
HSG.SRT.C.7	8	Explain and use the relationship between the sine and cosine of complementary angles
HSG.SRT.C.8	8	Use trigonometric ratios, *special right triangles*, and the Pythagorean Theorem to find unknown measurements of right triangles in applied problems Note: Examples should Including, but are not limited to *angles of elevation*, *angles of depression*, navigation, and surveying.

Domain: Circles

Cluster(s): 10. Understand and apply theorems about circles
11. Find arc lengths and areas of sectors of circles

HSG.C.A.1	10	Prove that all circles are similar  http://www.azed.gov/azcommoncore/files/2012/11/high- school-ccss-flip-book-usd-259-2012.pdf
HSG.C.A.2	10	Identify, describe, and use relationships among angles, *radii,* segments, lines, *arcs*, and *chords* as related to circles Note: Examples include but are not limited to the following: the relationship between central, inscribed, and circumscribed angles and their intercepted arcs; angles inscribed in a semi-circle are right angles; the radius of a circle is perpendicular to a *tangent line* of the circle at the point of tangency.
HSG.C.A.3	10	• Construct the *inscribed* and *circumscribed* circles of a triangle • Prove properties of angles for a quadrilateral inscribed in a circle
HSG.C.B.5	11	• Derive using similarity that the length of the *arc* intercepted by an angle is proportional to the *radius* • Derive and use the formula for the area of a *sector* • Understand the radian measure of the angle as a unit of measure Note: Connected to F.TF.1 (+)

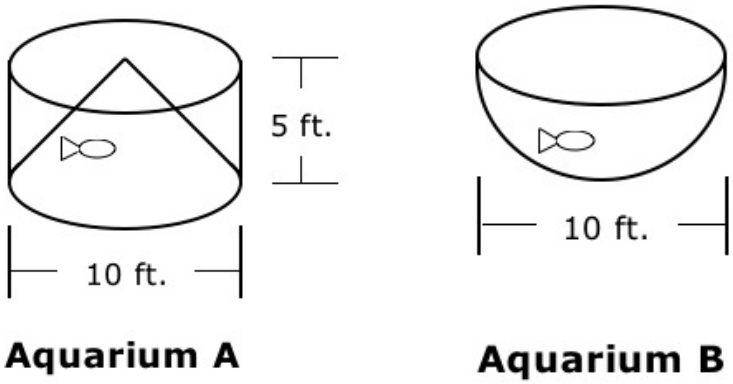
Domain: Expressing Geometric Properties with Equations

Cluster(s): 12. Translate between the geometric description and the equation of a conic section

13. Use coordinates to prove simple geometric theorems algebraically

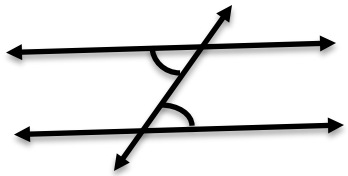
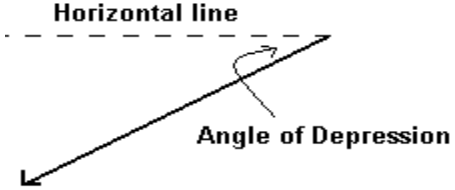
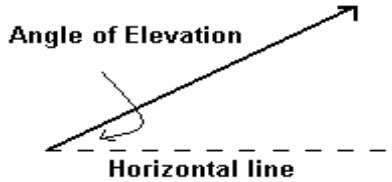
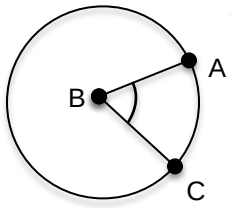
HSG.GPE.A.1	12	• Derive the equation of a circle of given center and radius using the Pythagorean Theorem • Complete the square to find the center and radius of a circle given by an equation Note: Students should also be able to identify the center and radius when given the equation of a circle and write the equation given a center and radius.
HSG.GPE.B.4	13	Use coordinates to prove simple geometric theorems algebraically For example: Prove or disprove that a figure defined by four given points in the coordinate plane is a rectangle; prove or disprove that the point (1, $\sqrt{3}$) lies on the circle centered at the origin and containing the point (0, 2).
HSG.GPE.B.5	13	• Prove the *slope* criteria for parallel and perpendicular lines • Use the *slope* criteria for parallel and perpendicular lines to solve geometric problems Note: Examples should include but are not limited to finding the equation of a line parallel or perpendicular to a given line that passes through a given point.
HSG.GPE.B.6	13	Find the *midpoint* between two given points; and find the endpoint of a line segment given the midpoint and one endpoint Note: An extension of this standard would be to find the point on a directed line segment between two given points that partitions the segment in a given ratio.
HSG.GPE.B.7	13	Use coordinates to compute *perimeters* of polygons and *areas* of triangles and rectangles Note: Examples should include, but are not limited using the distance formula and area of composite figures.

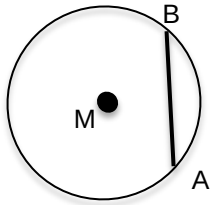
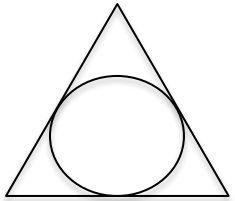
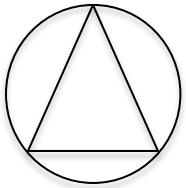
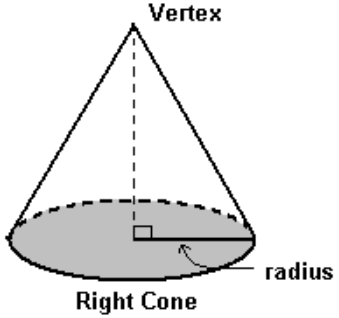
Domain: Geometric measurement and dimension
Cluster(s): 14. Explain volume formulas and use them to solve problems
15. Visualize relationships between two-dimensional and three-dimensional objects

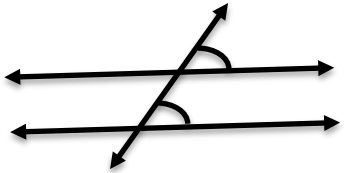
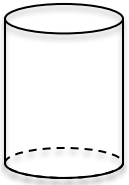
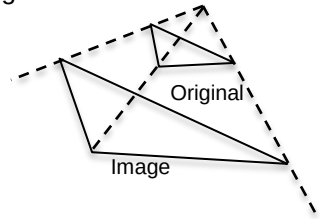
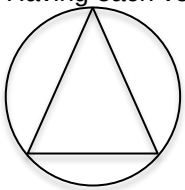
HSG.GMD.A.1	14	Give an informal argument for the formulas for the *circumference* of a *circle*, *area* of a *circle*, *volume* and *surface area* of a *cylinder*, *pyramid*, and *cone* For example: Use dissection arguments, *Cavalieri's principle*, and informal limit arguments.
HSG.GMD.A.2	14	(+) Give an informal argument using *Cavalieri's principle* for the formulas for the *volume* of a *sphere* and other solid figures  https://www.illustrativemathematics.org/content-standards/HSG/GMD/A/2/tasks/530
HSG.GMD.A.3	14	• Use *volume* formulas for *cylinders*, *pyramids*, *cones*, *spheres*, and to solve problems which may involve composite figures • Compute the effect on *volume* of changing one or more dimension(s) For example: How is the volume affected by doubling, tripling, or halving a dimension?
HSG.GMD.B.4	15	• Identify the shapes of two-dimensional *cross-sections* of three- dimensional objects • Identify three-dimensional objects generated by rotations of two-dimensional objects

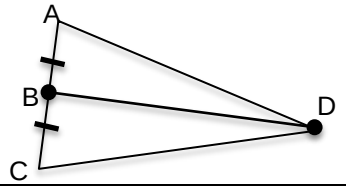
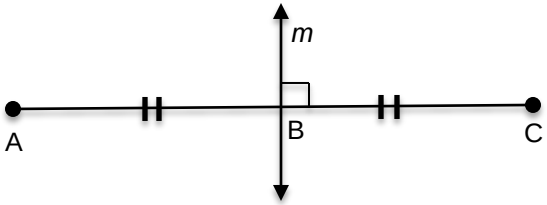
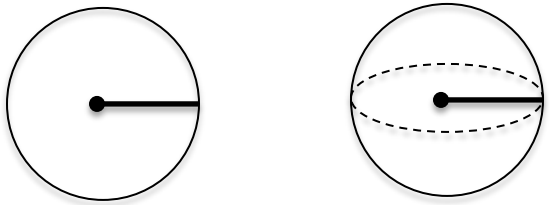
Domain: Modeling with Geometry
Cluster(s): 16. Apply geometric concepts in modeling situations

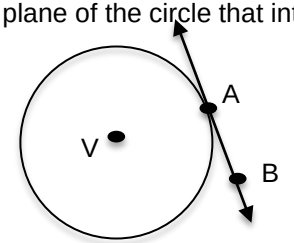
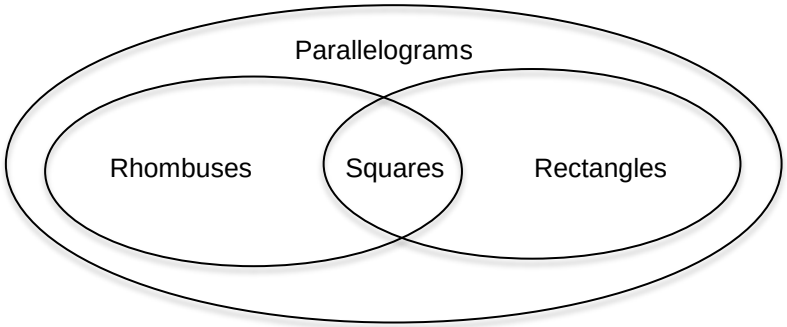
HSG.MG.A.1	16	Use geometric shapes, their measures, and their properties to describe objects (e.g., modeling a tree trunk or a human torso as a cylinder)
HSG.MG.A.2	16	Apply concepts of density based on area and volume in modeling situations (e.g., persons per square mile, BTUs per cubic foot)
HSG.MG.A.3	16	Apply geometric methods to solve design problems (e.g., designing an object or structure to satisfy physical constraints or minimize cost; working with typographic grid systems based on ratios)

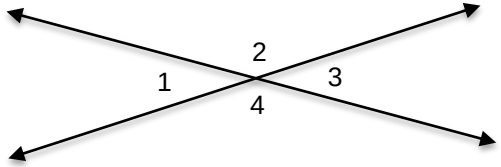
Alternate interior angles	Two angles that lie on opposite sides of a transversal between two lines that the transversal intersects 
Angle	Two noncollinear rays having a common endpoint
Angle of depression	The angle formed by a horizontal line and the line of sight of a viewer looking down 
Angle of elevation	The angle formed by a horizontal line and the line of sight of a view looking up 
Arcs	Two points on a circle and the continuous part of the circle between them 
Area	The measure of the size of the interior of a figure, expressed in square units
Cavalieri's principle	If two solids have the same cross-sectional area whenever they are sliced at the same height, then the two solids have the same volume
Center of a circle	The coplanar point from which all points of the circle are the same distance
Central angle	An angles whose vertex is the center of a circle and whose sides pass through the endpoints of an arc  ABC is a central angle of circle B.

Chord	A segment whose endpoints lie on the circle  $\overline{AB}$ is a chord on circle M.
Circle	The set of all points in a plane at a given distance from a given point
Circumference	The perimeter of a circle, which is the distance around a circle
Circumscribed (about a circle)	Having all sides tangent to the circle  The triangle is circumscribed about the circle.
Circumscribed (about a polygon)	Each vertex of the polygon lines on the circle  The circle is circumscribed about the triangle.
Complementary angles	Two angles (adjacent or nonadjacent) whose sum is 90 degrees
Conditional statements	A statement that can be expressed in 'if-then' form
Cone	A three dimensional figure with one circular base and a vertex 

Congruent	Identical in shape and size (angles, line segments, circles or polygons)
Contrapositive	The statement formed by exchanging and negating the hypothesis and conclusion of a conditional statement
Converse	The statement formed by exchanging the hypothesis and conclustion of a conditional statement
Corresponding (side or angle)	A side (or angle) of a polygon that is in the same position as a side (or angle) of a congrent or similar polygon
Corresponding angles	Two angles formed by a transversal intersecting two lines that lie in the same position relative to the two lines and the transversal 
Cross-section	A plane figured obtained by the intersection of a solid with a plane
Cylinder	A three dimensional figure with congruent, parallel bases 
Deductive reasoning	The process of showing that certain statements follow loginally from agree-upon assumptions and proven facts
Dilation	A nonrigid transformation that enlarges or reduces a geometric figure by a scale factor relative to a point 
Inductive reasoning	The process of observing data, recognizing patterns, and making generalizations about those patterns
Inscribed (in a circle)	Having each vertex on the circle  The triangle is inscribed in the circle.
Interior angle	An angle of a polygon that lies inside the polygon
Inverse statement	The statement formed by negating the hypothesis and conclusion of a conditional statement
Line	A straight, continuous arrangement of infinitelymany points extending forever in two directions
Line segment	Two points and all the points between them that are collinear with the two points

Median	A line segment connecting a vertex of a triangle to the midpoint of the opposite side  $\overline{DB}$ is the median of triangle ADC.
Midpoint	The point on the line segment that is the same distance from both endpoints;bisects the segment
Parallel lines (segments or rays)	Coplanar lines(segment or rays) that do not intersect
Parallelogram	A quadrilateral with both paris of opposite sides parallel
Perimeter	The sum of the lengths of the sides of a polygon; distance around
Perpendicular bisector	A line (segment or ray) that divides a line segment into two congruent parts and is perpendicular to the line segment  Line *m* is the perpendicular bisector of $\overline{AC}$.
Perpendicular lines (segments or rays)	Lines (segments or rays) that meet at 90^o angles
Plane	A flat surface that extends indefinitely along its edges; two-dimensional with a length and width, but no thickness
Point	A locatoin with no size or dimension
Polygon	A closed plane figure whose sides are segments that intersect only at their endpoints, with each segment intersecting exactly two other segments
Pyramid	A polyhedron consisting f a polygon base and triangular lateral faces that share a common vertex
Radius (circle or sphere)	A line segment from the center of a circle or sphere to a point on the circle or sphere 
Rectangle	A parallelogram with opposite sides congruent
Reflection	An isometry in which every point and its image are on opposite sides and the same distance from a fixed line
Regular polygon	A polygon with all sides congruent and all angles congruent
Rigid motion	A transformation that preserves size and shape; image congruent to original figure
Rotation	An isometry in which each point is moved by the same angle measure in the same direction along a circular path about a fixed point
Scale factor	The ratio of corresponding lengths in similar figures

Sector of a circle	The region between two radii and an arc of the circle 
Similarity	A transformation that preserves angles and changes all distances in the same ratio
Similar	Two figures are similar if and only if all corresponding angles are congruent and lengths of all corresponding sides are proportional
Slope	The ratio of the vertical change to the horizontal change between two points on a line
Special right triangles	A triangle whose angles are either 30-60-90 degrees or 45-45-90 degrees
Sphere	The set of all points in space at a given distance from a given point
Supplementary angles	Two angles (adjacent or nonadjacent) whose sum is 180 degrees
Surface area	The sum of the areas of all of the surfaces of a solid
Tangent line	A line in a plane of the circle that intersects the circle in only one point  $\overline{AB}$ is a tangent of circle V.
Theorem	A conjecture that has been proved within a deductive system
Transformation	A rule that assigns to each point of a figure another poin in the plane, called its image
Translation	An isometry in which each point is moved by the same translation vector
Transversal	A line that intersect two or more other coplanar lines
Trapezoid	A quadrilateral with at least one pair of opposite sides parallel
Venn diagram	A concept map of overlapping circles or ovals that shows the relationships among members of different sets 

Vertical angles	Nonadjacent, nonoverlapping congruent angles formed by two intersecting lines; share a common vertex  $\angle 1$ and $\angle 3$ are vertical angles. $\angle 2$ and $\angle 4$ are vertical angles.
Volume	A measure of the amount of space contained in a solid, expressed in cubic units